On Semistructured Data and Lingustics

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Overview

- Introduction and Motivation
- Semistructured Data
- Query Languages
- Intermediate Conclusion
- Secondary Edges
- Regular Queries of Context-Sensitive Relations
- MSO Queries
- Lifting
- Reconstruction
- Conclusion and Outlook

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Introduction and Motivation

The Two Cultures

- exchange of information
- The Web
  - decomposition into units: files
  - structure (internal and between documents)
- Databases
  - relational database schemata or entity relationship diagrams
  - logical or abstract vs. physical implementation
  - concerns
    - query languages
    - mechanisms for concurrency control and for recovery
    - efficient implementation

Why do we have to bridge the gap?

- TV station A publishes data about political parties
  - stored: relational database
  - output is generated on demand in HTML via the formatting of the results of an SQL query
- Party B wants to analyze A's data
  - access only to A's HTML pages
  - write software for the analysis (parsing of HTML)
- Problems:
  - brittle
  - need to download many pages to get at the desired information

What we have . . .

- The Web
  - global infrastructure and standards for document exchange
  - presentation of hypertexts
  - user interfaces for document retrieval (IR techniques)
  - XML (data with structure)
- Databases
  - storage techniques and query languages with efficient methods for large amounts of (rigidly) structured data
  - data models and methods for structuring data
  - mechanisms for maintaining integrity and consistency
  - new model: semistructured data (relaxation of the demands on the structuring)

Basic Architecture

- Databases: client/server architecture
  - client issues a query
  - processing, optimization and execution
  - answer is returned by the server
- Web: “multitier” architecture
  - lowest level: data sources (database or file servers, legacy systems, “anything that produces data”) in a common logical data model and a common format (XML)
  - highest level: user interfaces or analysis software
  - middleware: transforms, integrates, adds data (data warehousing, mediator systems)
  - queries: translation, processing, translation

Data Abstraction

- databases: three levels
  - physical
  - logical
  - external
- web
  - no distinction between physical and logical layer
  - is there a reason why we should make this distinction?
  - Example: database can be a single XML file, or a compressed version, or a relational database, or . . .

➪ XML as both physical and logical representation

Data Diversity

- Heterogeneous data
- How is the data conceptually represented?
- Example: addresses – sorted by name? sorted by town?
- Reconciliation at the logical level is possible

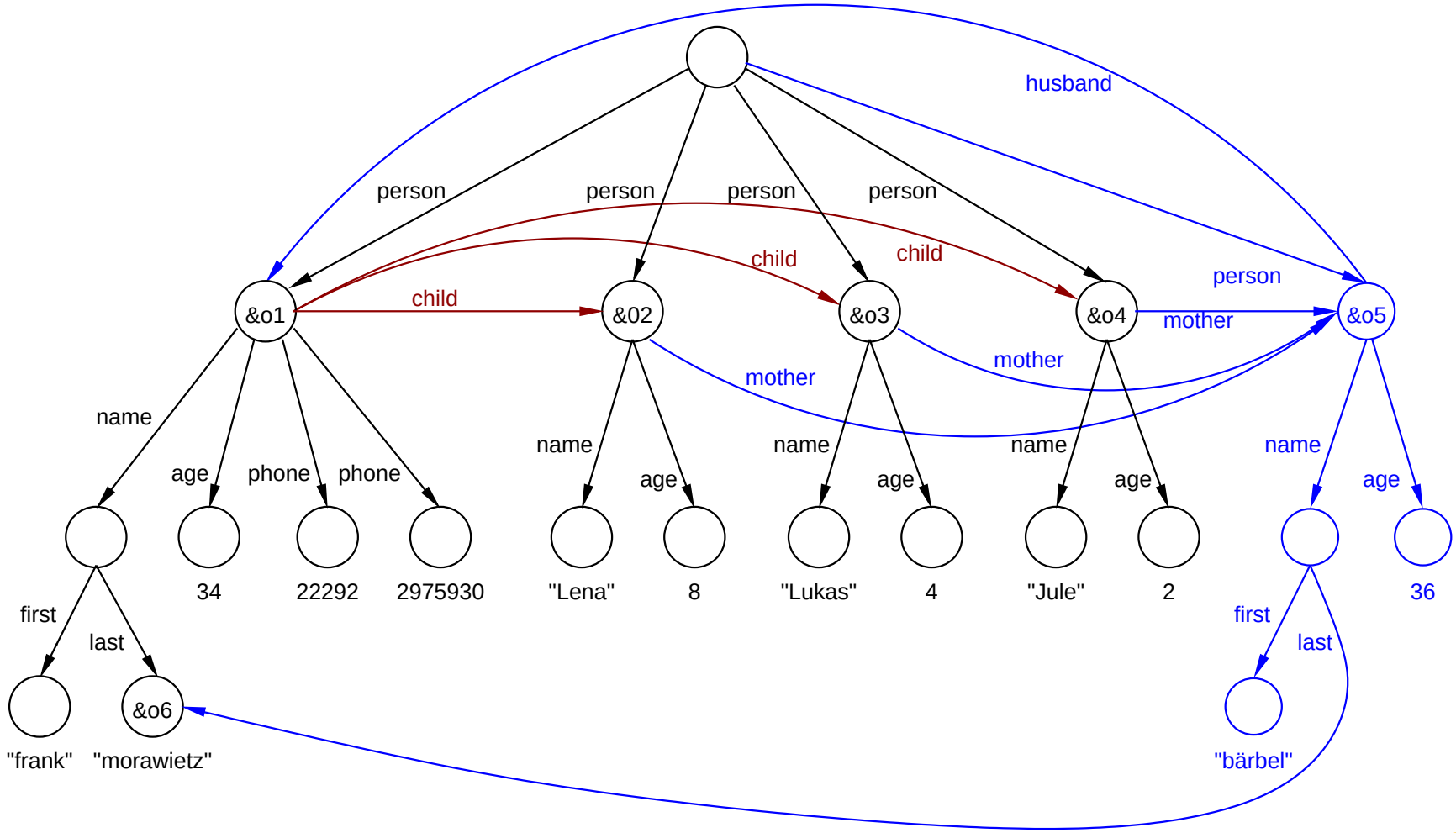
➪ XML for the logical level

Research Context

- **Semistructured Data**
  - incompleteness and irregularity of data
  - self-describing
  - integration of several kinds of data
  - no relational schemata, but structured
- **XML**
  - universal format for data exchange
  - data model: labeled graphs/trees
  - query languages

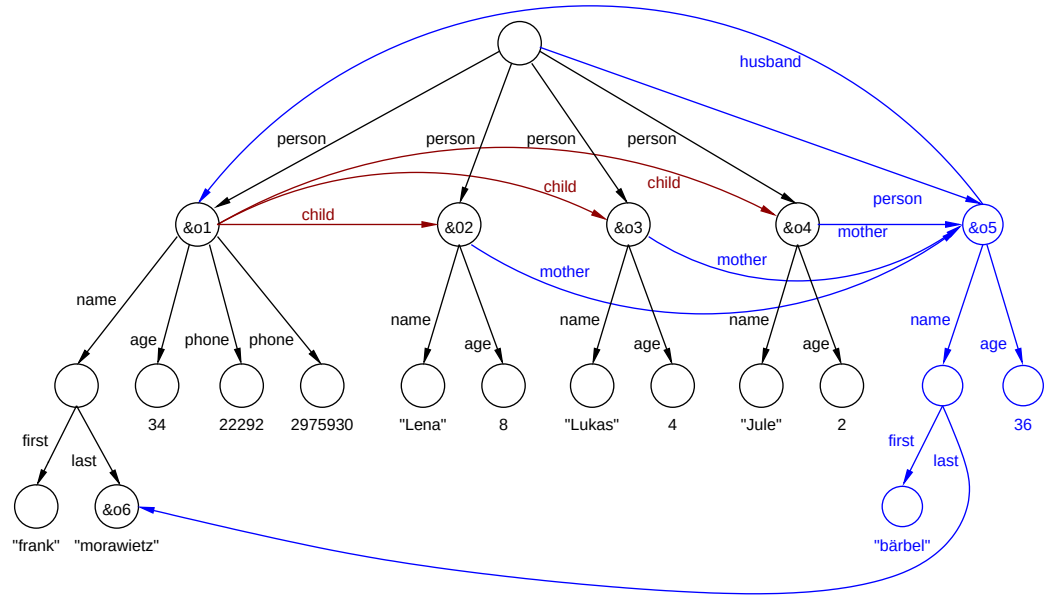
Semistructured Data

The Data Model: Syntax



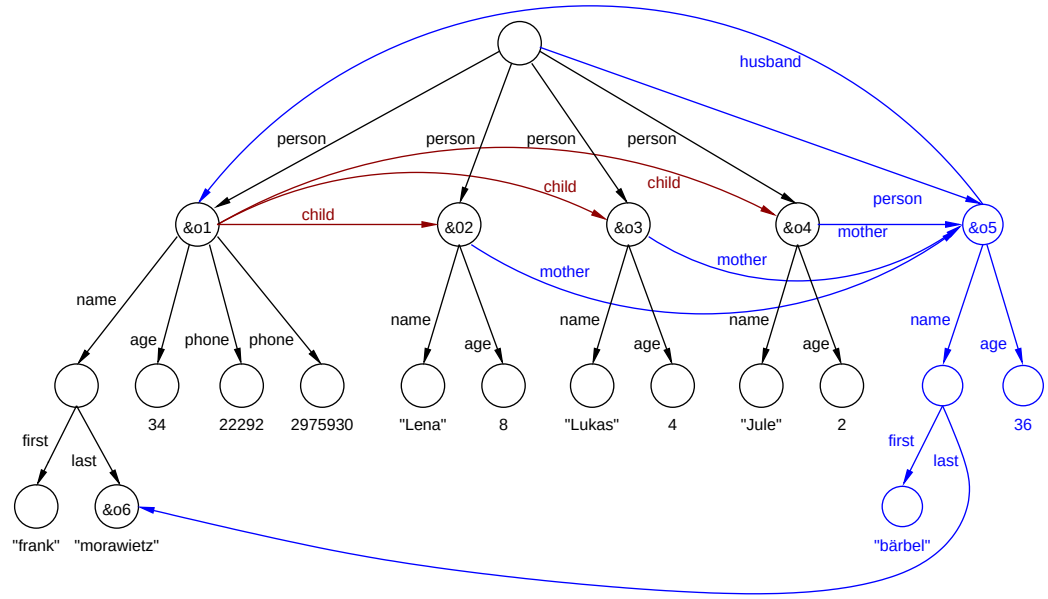
The Data Model: Syntax

```
{name: "Lena", age: 8}
```



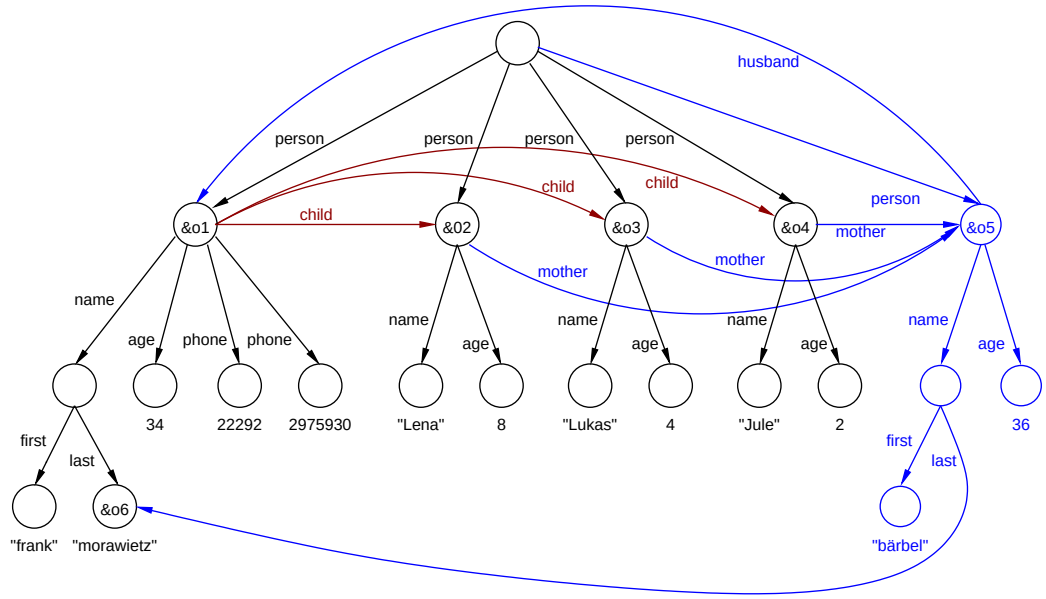
The Data Model: Syntax

```
{name: {first: "frank", last: "morawietz"},
age: 34,
phone: 22292}
```



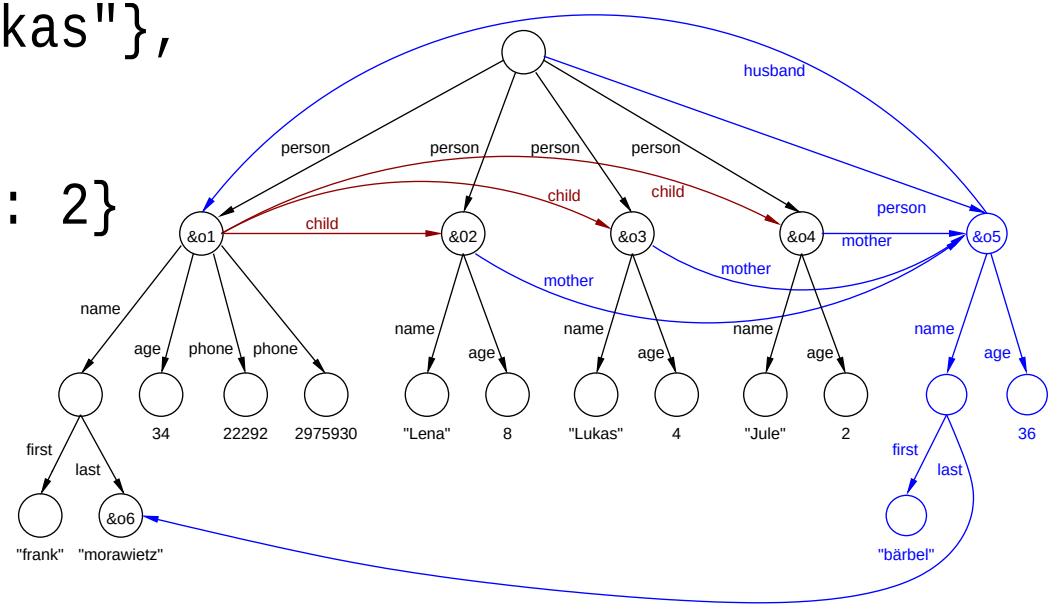
The Data Model: Syntax

```
{name: {first: "frank", last: "morawietz"},
age: 34,
phone: 22292,
phone: 2975930}
```



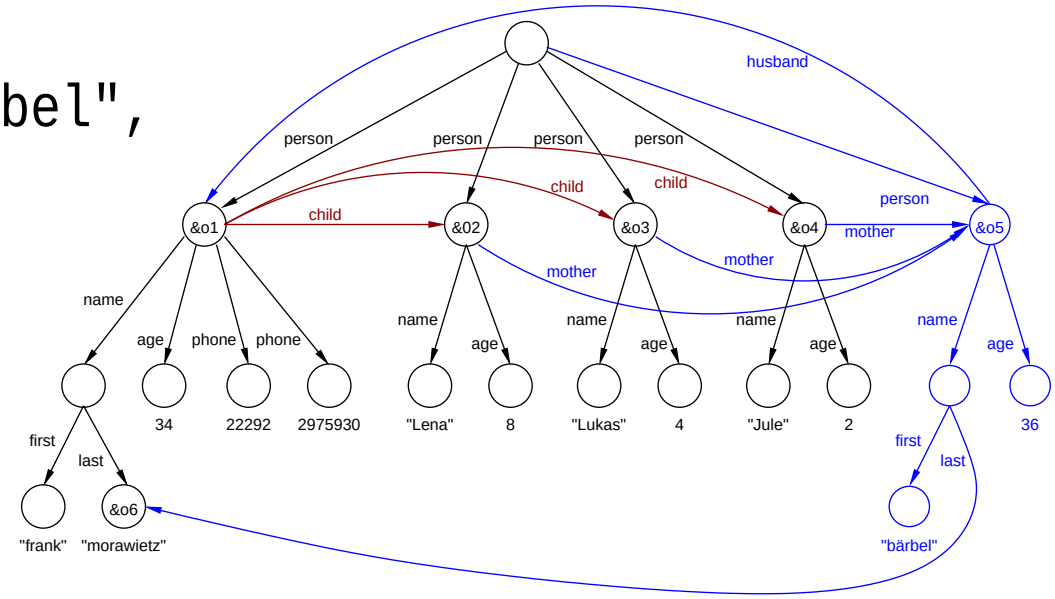
The Data Model: Syntax

```
{
person: {name: "Lena", age: 8},

person: {age: 4, name: "Lukas"},

person: {name: "Jule", age: 2}
}
```



The Data Model: Syntax

```
{
person:
    &o2{name: "Lena", age: 8, mother: &o5 },
person:
    &o5{name: {first: "Bärbel",
               last: &o6},
       age: 36,
       husband: &o1
      },

...
}
```



The Data Model: Syntax

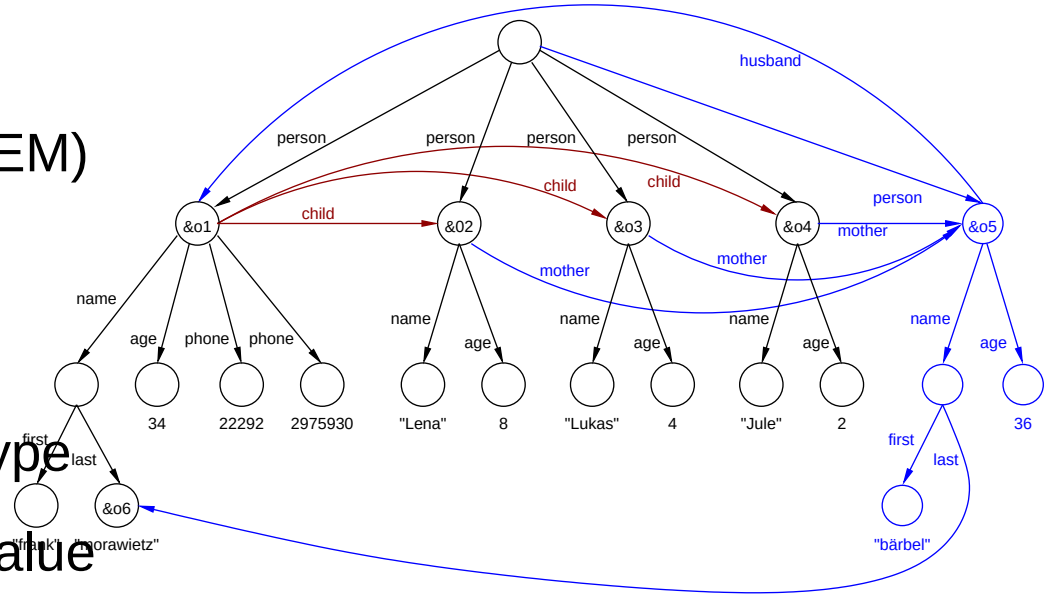
- Convention: implicit OIDs
- `{a: {b: 3}, a: {b: 3}}`
- coding of relational databases (e.g., via rows)
- edge labeled graph
- ⇨ Object Exchange Model (OEM)

label: string

oid: object identifier

type: complex or a specific type

value: list of OIDs or atomic value



Semistructured Data and XML

```
{ person: {name: {first: "frank", last: "morawietz"},
           age: 34,
           phone: 22292}}
```

```
<person>
  <name>
    <first> frank </first>
    <last> morawietz </last>
  </name>
  <age> 34 </age>
  <phone> 22292 </phone>
</person>
```

Semistructured Data in Linguistics

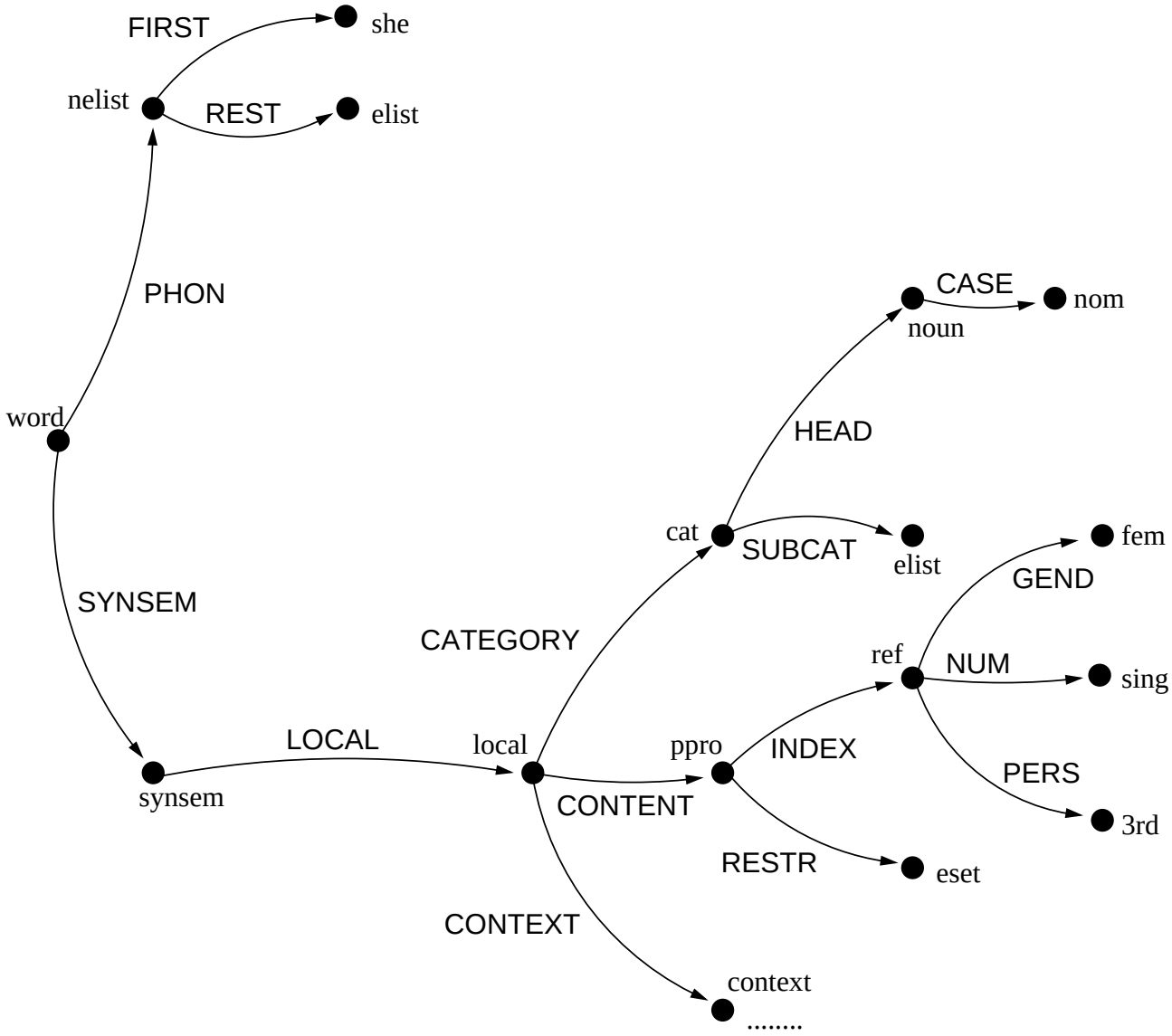
Linguistic Databases: collections of linguistic data

- relational databases (e.g., typological databases)
- lexicons
- corpora (e.g., books, newspaper articles, transcriptions, . . .)

Semistructured Data in Linguistics

Language	WO	Prep	Case	Adj	...
German	SOV	←	Acc	←	...
English	SVO	←	Acc	←	...
Warao	SVO?	→	Acc	→	...

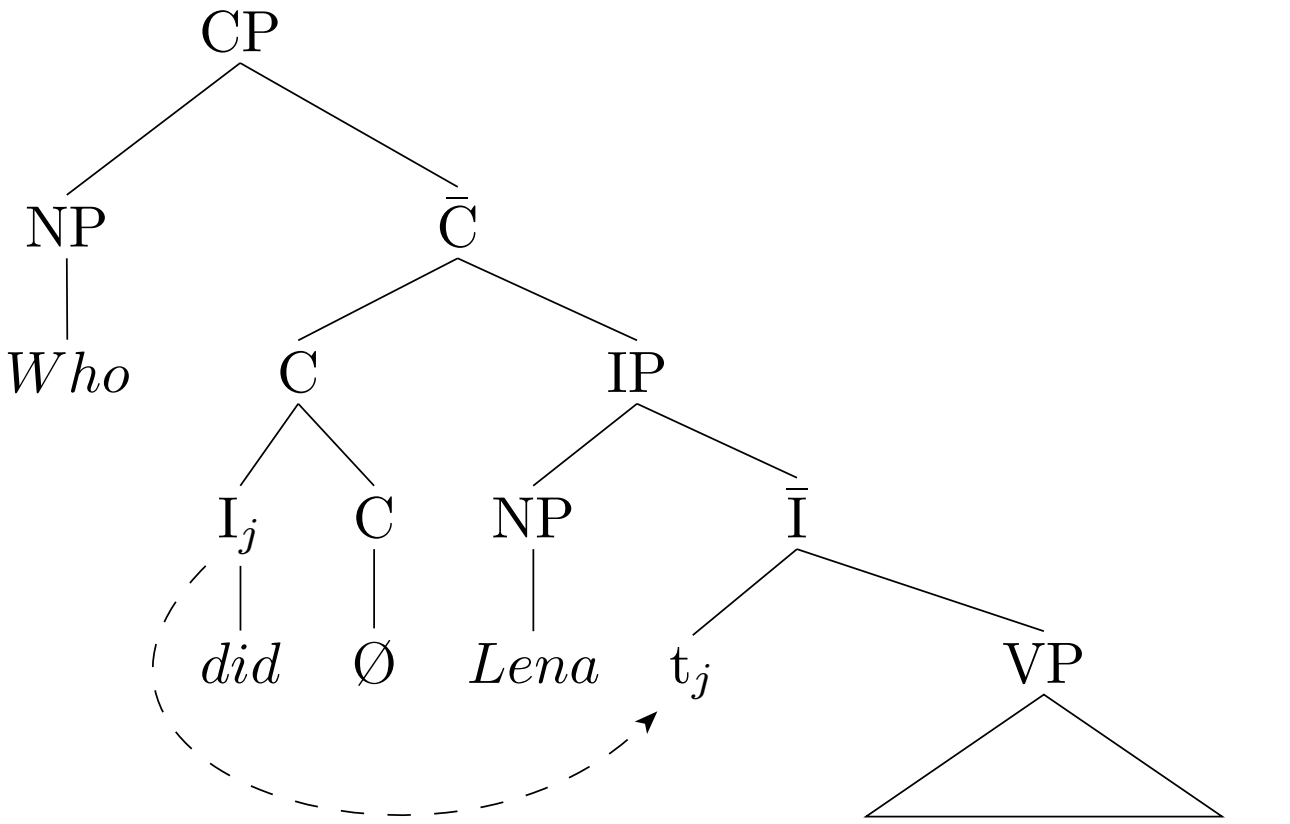
Semistructured Data in Linguistics



Semistructured Data in Linguistics

$$
\begin{bmatrix}
\textit{word} \\
\textit{phon} & \begin{bmatrix} \textit{nelist} \\ \textit{first}\ \textit{she} \\ \textit{rest}\ \textit{elist} \end{bmatrix} \\
\textit{synsem} & \begin{bmatrix} \textit{synsem} \\ \textit{local} & \begin{bmatrix} \textit{local} \\ \textit{category} & \begin{bmatrix} \textit{cat} \\ \textit{head} & \begin{bmatrix} \textit{noun} \\ \textit{case}\ \textit{nom} \end{bmatrix} \\ \textit{subcat}\ \textit{elist} \end{bmatrix} \\ \textit{content} & \begin{bmatrix} \textit{ppro} \\ \textit{index} & \begin{bmatrix} \textit{ref} \\ \textit{gend}\ \textit{fem} \\ \textit{num}\ \textit{sing} \\ \textit{pers}\ \textit{3rd} \end{bmatrix} \\ \textit{restr}\ \ \textit{eset} \end{bmatrix} \\ \textit{context} & \begin{bmatrix} \textit{context} \\ \ldots\ \ldots \end{bmatrix} \end{bmatrix} \end{bmatrix}
\end{bmatrix}
$$

Semistructured Data in Linguistics



Who did Lena invite to her birthday party?

Semistructured Data in Linguistics

Penn Treebank II:

$(TOP\ (S\ (NP\text{-}SBJ$ my best friend$)$
$(VP$ gave
$(NP$ me$)$
$(NP$ chocolate$)$
$(NP\text{-}TMP$ yesterday$))$
$.))$

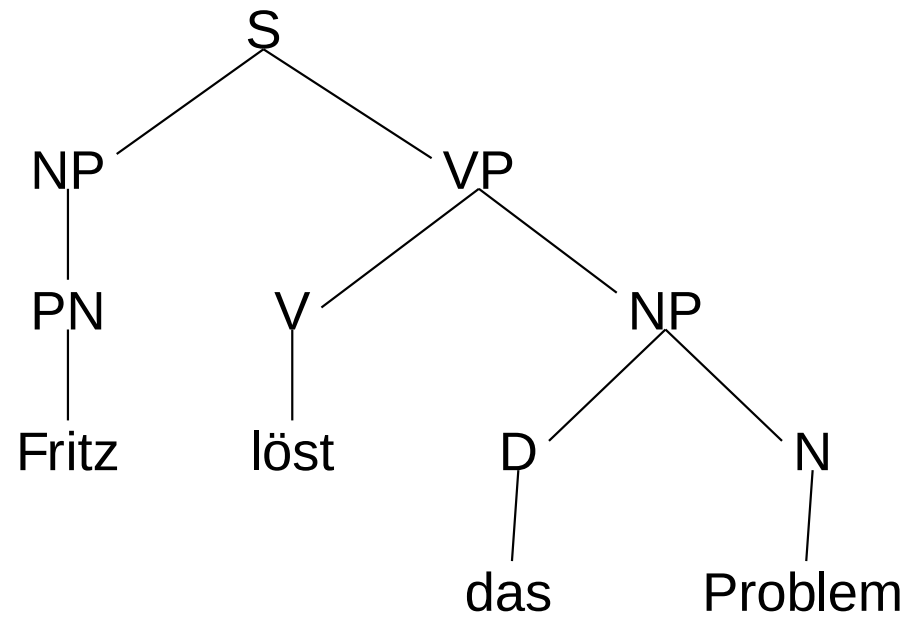
Annotation

Additional information to the “raw” data

- meta information
- document structure
- part-of-speech tags
- syntactical analysis
- specific syntactic or semantic phenomena
- …

Core XML

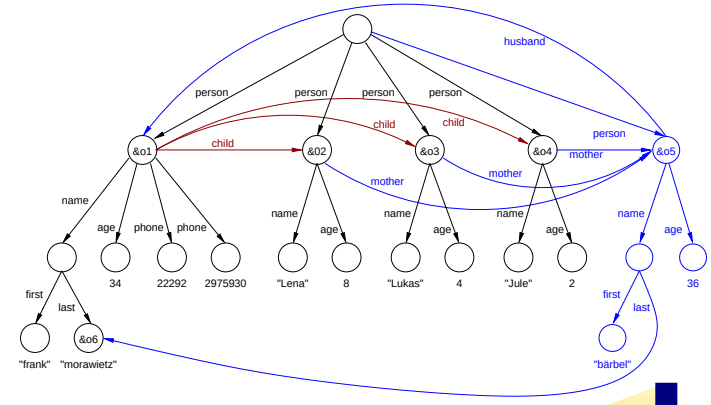
```
<S>
   <NP>
      <PN> Fritz </PN>
   </NP>
   <VP>
      <V> löst </V>
      <NP>
         <D> das </D>
         <N> Problem </N>
      </NP>
   </VP>
</S>
```



Query Languages

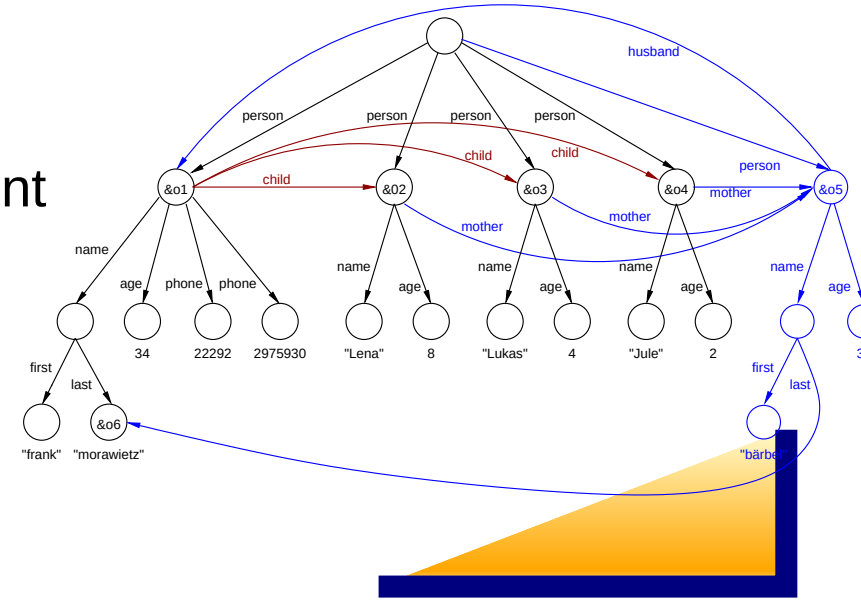
A simple Query Language

- path expressions (`"person.name.first"` yields the set of nodes with labels `"frank"` and `"bärbel"`)
- path expressions result in sets of nodes
- paths via properties: regular expressions (including a wild card)
  - on the alphabet of edge labels (`"person._*.name"`)
  - on the alphabet of the characters of edge labels (`(((n|N)ame)(s)?)`)
- Query syntax based on Lorel/UnQL
  - ```
    select firstname:  X
    from person.name.(first)?  X
    ```
  - ```
    select firstname:  X
    from person.name X
    where "frank" in X.first
    ```



More Complex Queries

- ```
  select person:  X
  from person X
       X.age Y
  where Y < 30
  ```
- ```
  select person:  {firstname:  Y, age:  Z}
  from person X
       X.name.(first)?  Y
       X.age Z
  ```
- nested queries in the `select` statement
- natural joins



Syntactic Sugar for Queries

- default labels
- path expressions in the `select` clause
- equality predicates
- coercion
- relation names as variables
- label and path variables

Desiderata for Query Languages

- Expressive Power
  - operations (comparable to, e.g., SQL)
  - restructuring
  - completeness
- Semantics
  - query transformation
  - query optimization
- Compositionality: output can be used as input of a query
  - same data model
  - referential transparency
- Structure Consciousness
- Program Manipulation

Reintroducing Structure: Why?

- Ignoring knowledge of types loses information
- Regularities facilitates querying.
- Protection against incorrect updates.
- Simplify access and improve performance.
- Optimization of the physical representation.
- Mixing querying structured and semistructured data.

- Maybe hidden from the user?

Query Languages for XML

- combine XML syntax with query languages presented previously
- XML-QL, XQuery, XSL(T)
- Problems:
  - Order
  - infinite domain of attributes

Advanced Topics

- first-order interpretation
- reconstruction
  - Datalog
  - Graphical Languages: GraphLog, Complete Answer Aggregates
  - Structural Recursion
  - Bisimulation
- Strudel/StruQL

TGrep (Penn Treebank)

`NP`	anything called just `NP`
`TOP`	anything called `TOP`. Since every sentence is enclosed in `TOP` brackets, this matches everything.
`VP < NP`	`VP` immediately dominates an `NP`
`VP <3 NP`	`VP` has `NP` as its 3rd child
`NP > VP`	an `NP` that has a `VP` parent
`VP << dog`	`VP` contains the word `dog` (`VP` dominates `dog`, directly or indirectly)
`S \!<< dog`	`S` does not contain the word `dog`
`NP $..  NP`	two `NP`s have a common parent (first `NP` precedes a sister also called `NP`)
`/^NP/`	`//` encloses a grep-style regular expression

TGrep (Penn Treebank)

```
S <1 /^NP/ < (VP < (NP $..  NP))
```

Get all Ss that start with an NP (not necessarily the subject) and that dominate a VP that in turn has two NP children – in other words, sentences with what might be double-object VPs.

TGrep2

- Rather than simply having a set of required relationships and a set of prohibited relationships, nodes can have full boolean expressions of relationships to other nodes.
- Nodes can be given unique labels and may then be referred to by those labels in the pattern specification or in selecting trees for printing.
- Patterns are no longer restricted to simple tree architectures. The use of node labels and segmented patterns allows links in a pattern to form back-edges as well, permitting cycles of links.
- Customizable output formats allow a variety of information to be reported in a flexible manner.
- Multiple search patterns may be specified and one can retrieve the first subtree matching any pattern, the first subtree matching each pattern, or all subtrees matching all patterns.
- Subtrees can be reported using a code rather than by printing the whole structure. The trees themselves can later be retrieved using the codes.
- A variety of new links have been added and the immediately-precedes link now has a more conventional meaning.
- TGrep2 corpus files are substantially smaller than tgrep corpora.

FXGrep (Neumann/Seidl)

- XML input as a document forest representing the logical structure
  - A text node represents one or more adjacent pieces of character data.
  - A PI node represents a processing instruction and is annotated with the target name of the processing instruction. It has a single child: a text node giving the text of the processing instruction.
  - An element node represents an element and is annotated with the element type and the attributes specified for the element. The children of an element node are a sequence of nodes constructed from the content of the element.
- query implemented by consecutive runs of two forest automata (for simple queries a single run suffices)

FXGrep: Pattern Language

`/doc/section/title`	paths
`/doc//title`	descendant
`//title`	descendant anywhere
`//(section/||subsection/)title`	or
`//title/"airplane"`	content
`//title[toc='yes']`	attributes
`//section[_ subsection _]/title`	titles of sections with subtitles
`a[_ b#c _]/d`	context qualifier

Alternatively with grammar patterns.

FXGrep: Pattern Language

```
//section[!_ subsection _][_(//footnote/'automata')_]/title
```

The titles of all sections that do not have a child of type `subsection` and that do have a descendant of type `footnote` which contains the text `automata`.

FXGrep: Theory

- words are monadic trees
- ranked trees are terms with internal nodes being operators and leaves being constants
- unranked trees represent document structure, nodes are elements
- unranked trees can be represented by ranked trees, but not uniquely
- trees can be specified with grammars, regular expressions, formulas in fixpoint or monadic second-order logic, automata
-

specification	automaton
grammar	polynomial
regular expressions	polynomial
fixpoint formulas	exponential
MSO formulas	non-elementary

- Nice animated slides by Helmut Seidl (Uni Trier)

Tree Automata

- ranked case
  - work bottom-up from the frontier to the root
  - transitions according to the states of the daughters and the label of the mother
  - a tree is accepted if it halts in a final state at the root
- unranked case
  - additional word automaton to recognize the sequence of labels of the daughters
- variations
  - extension to “hedges” (Murata, Brüggemann-Klein, Wood)
  - tree-walking automata navigate through the tree
  - caterpillars (Brüggemann-Klein, Herrmann, Wood)
  - pushdown tree automata (Neumann/Seidl)

Query Automata (Neven/Schwentick)

- two-way tree automata + a selection function
  - partition state-alphabet symbol pairs in U and D
  - always works on a cut of the tree
  - down transition: a node is replaced by its children
  - up transition: all children of a node are replaced by the node
  - ➪ new states for all nodes in the cut
- query: the set of nodes selected by the two-way tree automaton via the selection function
- Expressivity: unary MSO patterns
- Extension to the unranked case possible, but complicated

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Intermediate Conclusion

Fundamental Problem

Scylla (Expressivity)

vs.

Charybdis (Undecidability)

Fundamental Problem II

classical	**semistructured**
RA	“Automata”
FO	MSO
SQL	Tree Description language with regular path descriptions

Secondary Edges

Cross-serial dependencies

(. . . weil) der Karl die Maria dem Peter den Hans schwimmen lehren helfen liess

German: Dijck-Sprache—CF

(. . . omdat) Karel Marie Piet Jan liet helpen leren zwemmen

Dutch: $a^n b^n$—CF

(. . . wil) mer de maa em chind lönd hälffe schwüme

Cross-serial dependencies

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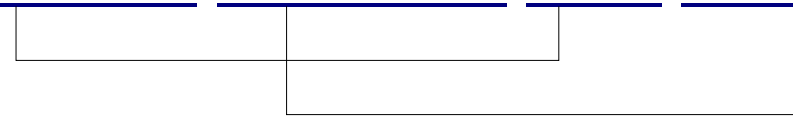

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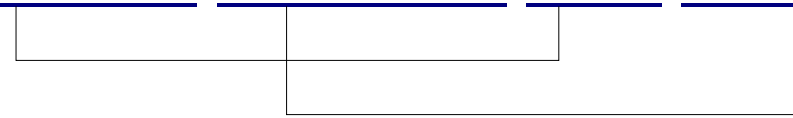


Swiss-German: $a^n b^m c^n d^m$—Non-CF

Regular Query of context-sensitive relations

Overview: Two-Step Approach

- Assumption: there exists a description of the context-sensitive phenomena (e.g., via a context-free tree grammar)
- database query
  - MSO query into the treebank
    - ➪ set of possible answers/trees
  - lifting of the grammar and the possible answer trees
    - ➪ tree automaton representing the grammar and a set of lifted candidate trees
  - the automaton filters the lifted candidate trees
    - ➪ set of lifted answer trees
- reconstruction of the intended trees via MSO-transduction or macro tree transducer
  - ➪ set of intended answer trees

Context-Free Tree Grammars

A context-free tree grammar Γ is a 5-tuple $\langle \Sigma, \mathbf{F}, S, \mathbf{X}, \mathbf{P} \rangle$, with

$\Sigma, \mathbf{F}$	ranked alphabets of *inoperatives* and *operatives*;
$S \in \mathbf{F}$	start symbol;
$\mathbf{X}$	a set of variables; and
$\mathbf{P}$	a set of productions.

Context-Free Tree Grammars

The productions $p \in \mathbf{P}$ are of the form

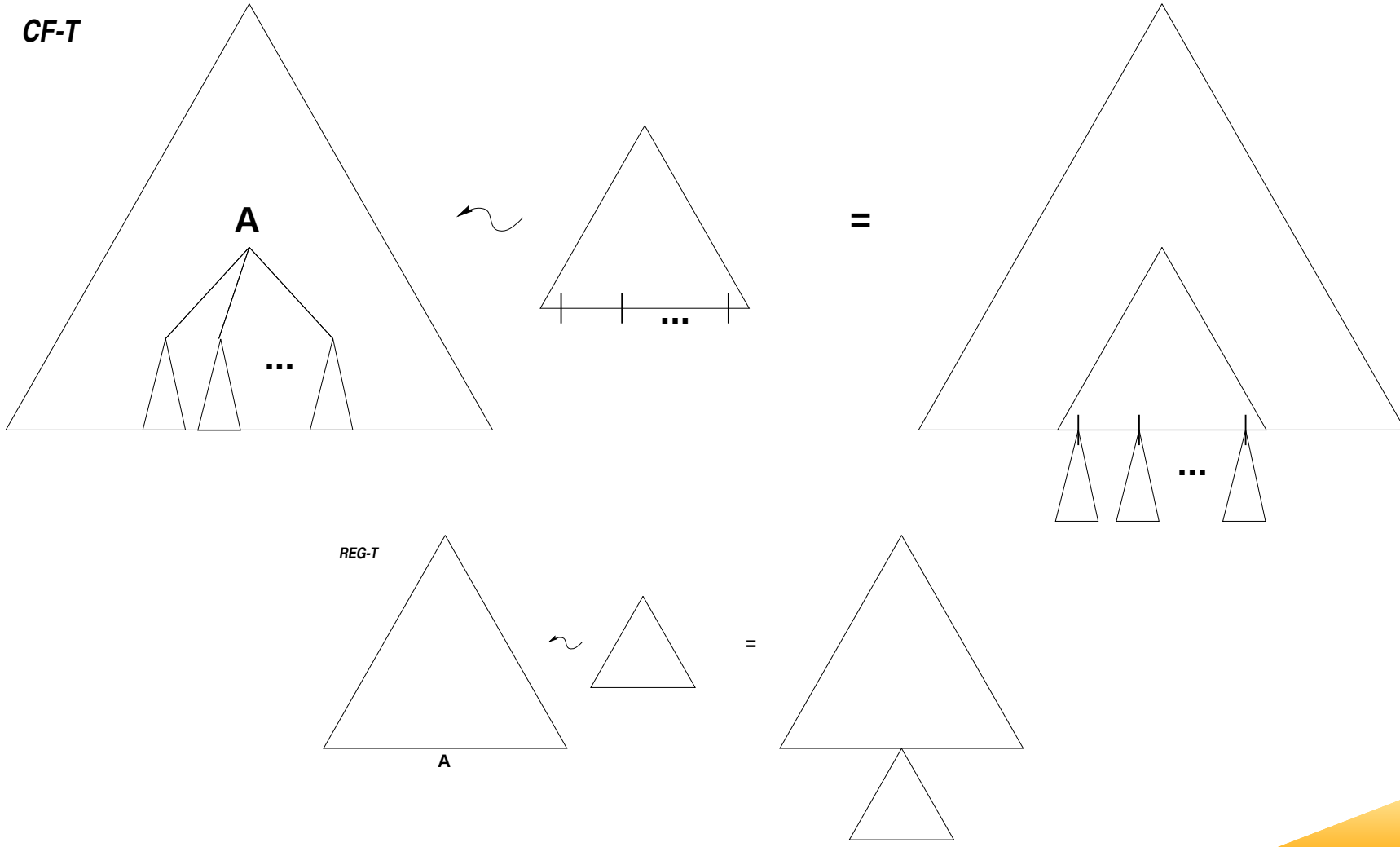
$$F(x_1, \cdots, x_n) \longrightarrow t$$

with $F \in \mathbf{F}$, $x_1, \cdots, x_n \in \mathbf{X}$, and t is a term over $\mathbf{\Sigma} \cup \mathbf{F} \cup \{x_1, \cdots, x_n\}$.

An application of a rule $F(x_1, \cdots, x_n) \longrightarrow t$ "rewrites" a subtree rooted in F as the tree t with its respective variables substituted by F's daughters.

Tree grammars with $\mathbf{F}_n = \emptyset, n \neq 0$, are called *regular*.

Context-Free Tree Grammars



A CFTG for $a^n b^m c^n d^m$: $\mathcal{G}$

$$\Sigma_0 = \{\varepsilon, a, b, c, d\} \qquad \Sigma_2 = \{\bullet\}$$

$$\mathbf{F}_0 = \{S\} \qquad \mathbf{F}_4 = \{F\}$$

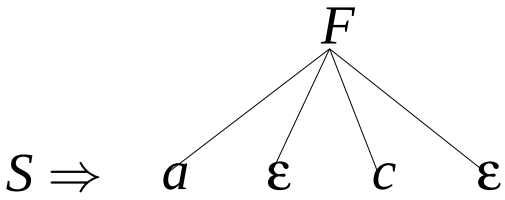
$$S = S \qquad \mathbf{X} = \{x_1, x_2, x_3, x_4\}$$

$$\mathbf{P} = \left\{ \begin{array}{rcl} S & \longrightarrow & \varepsilon \\ S & \longrightarrow & F(a, \varepsilon, c, \varepsilon) \\ S & \longrightarrow & F(\varepsilon, b, \varepsilon, d) \\ F(x_1, x_2, x_3, x_4) & \longrightarrow & F(\bullet(a, x_1), x_2, \bullet(c, x_3), x_4) \\ F(x_1, x_2, x_3, x_4) & \longrightarrow & F(x_1, \bullet(b, x_2), x_3, \bullet(d, x_4)) \\ F(x_1, x_2, x_3, x_4) & \longrightarrow & \bullet(\bullet(\bullet(x_1, x_2), x_3), x_4) \end{array} \right\}$$

An example derivation of *aabccd*

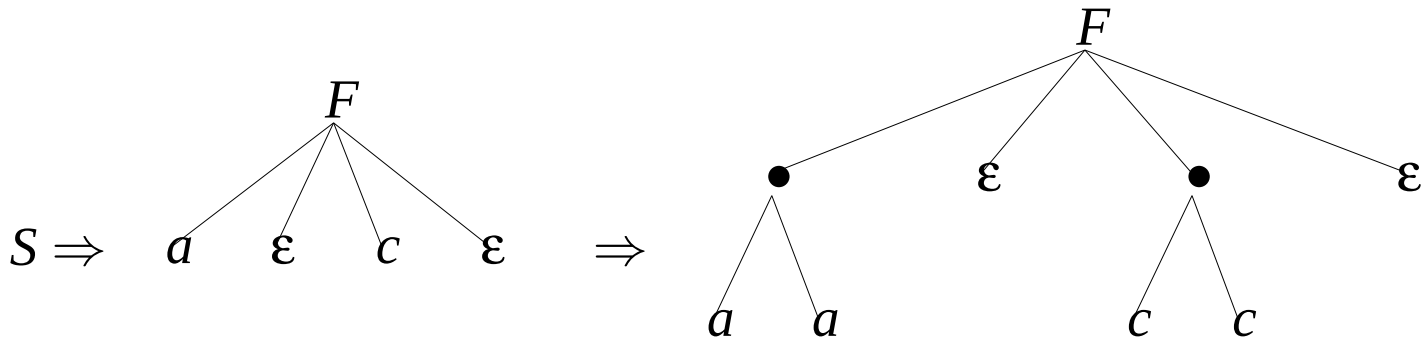
S

An example derivation of *aabccd*



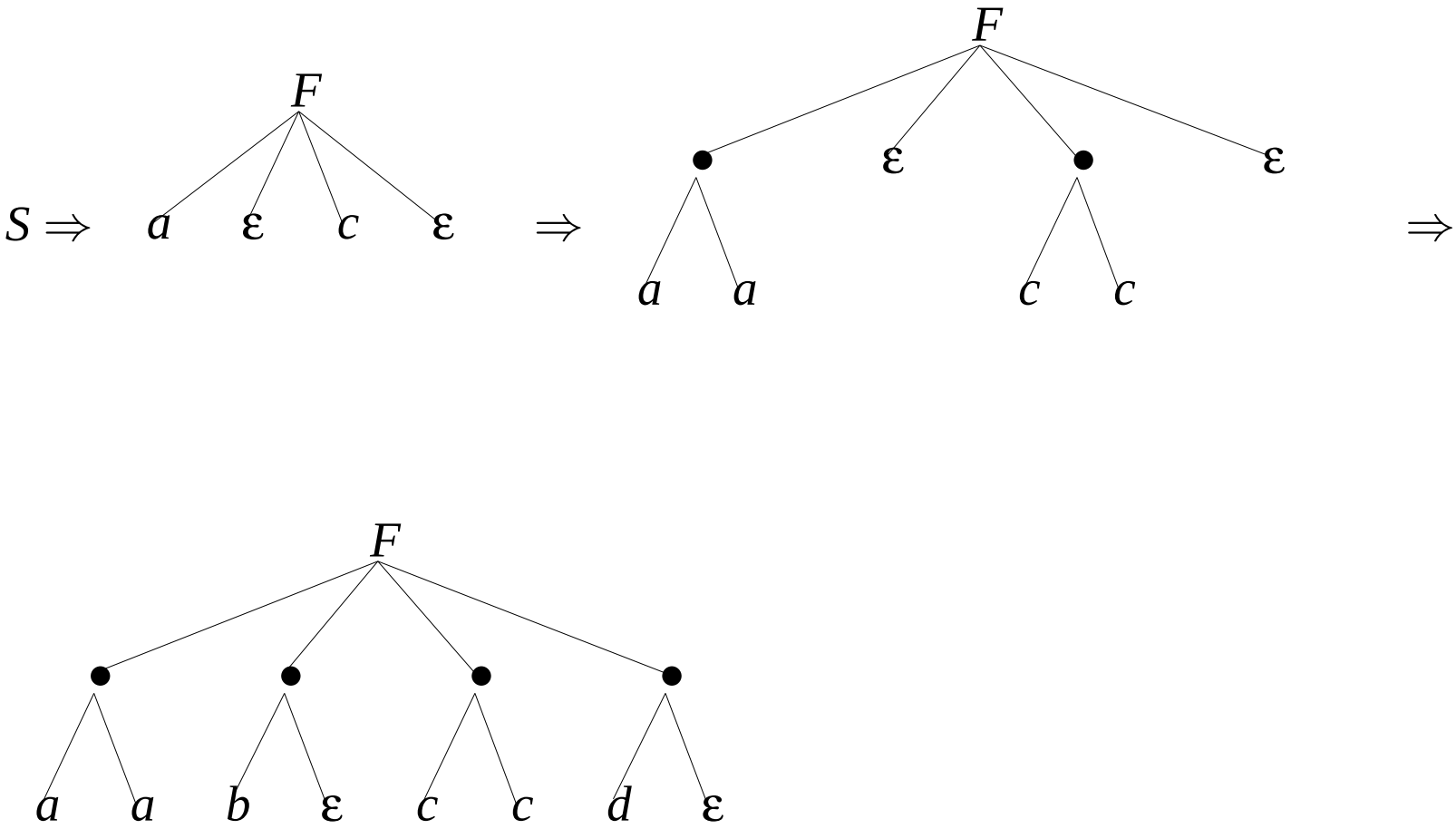
$S \longrightarrow F(a, \varepsilon, c, \varepsilon)$

An example derivation of *aabccd*



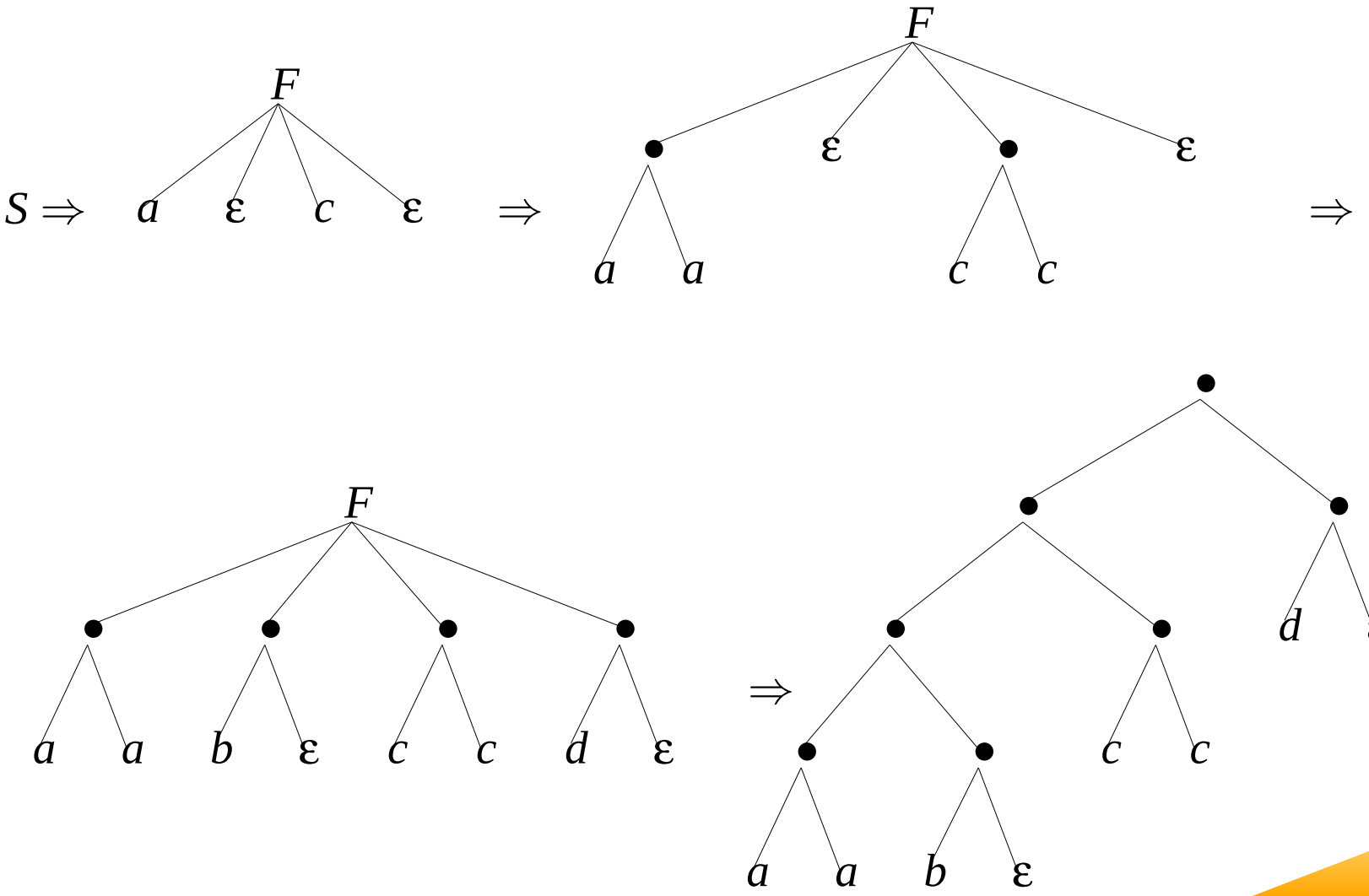
$F(x_1, x_2, x_3, x_4) \longrightarrow F(\bullet(a, x_1), x_2, \bullet(c, x_3), x_4)$

An example derivation of *aabccd*



$F(x_1, x_2, x_3, x_4) \longrightarrow F(x_1, \bullet(b, x_2), x_3, \bullet(d, x_4))$

An example derivation of *aabccd*



$F(x_1, x_2, x_3, x_4) \longrightarrow \bullet(\bullet(\bullet(x_1, x_2), x_3), x_4)$

MSO as a Query Language

MSO Queries

$$Q = \{t \models \varphi_r\}$$

$r \overset{def}{\Longleftrightarrow}$ regular tree expression

$\varphi_r \overset{def}{\Longleftrightarrow}$ translation of r into an MSO formula

Example:

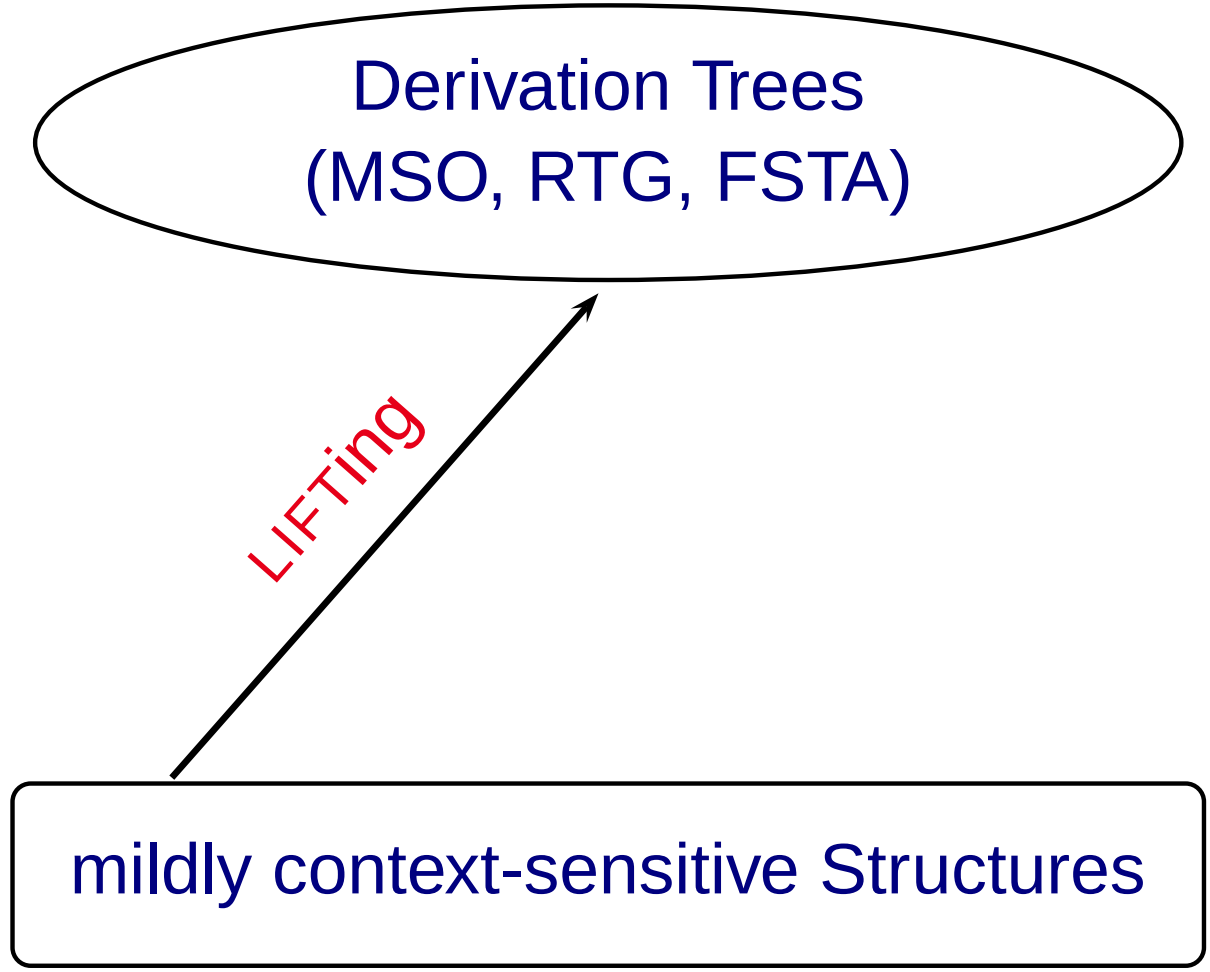
$$(NP_{akk} \cup NP_{dat})^* \cdot \_ \cdot (V_{akk} \cup V_{dat})^*$$

Overview Two-Steps

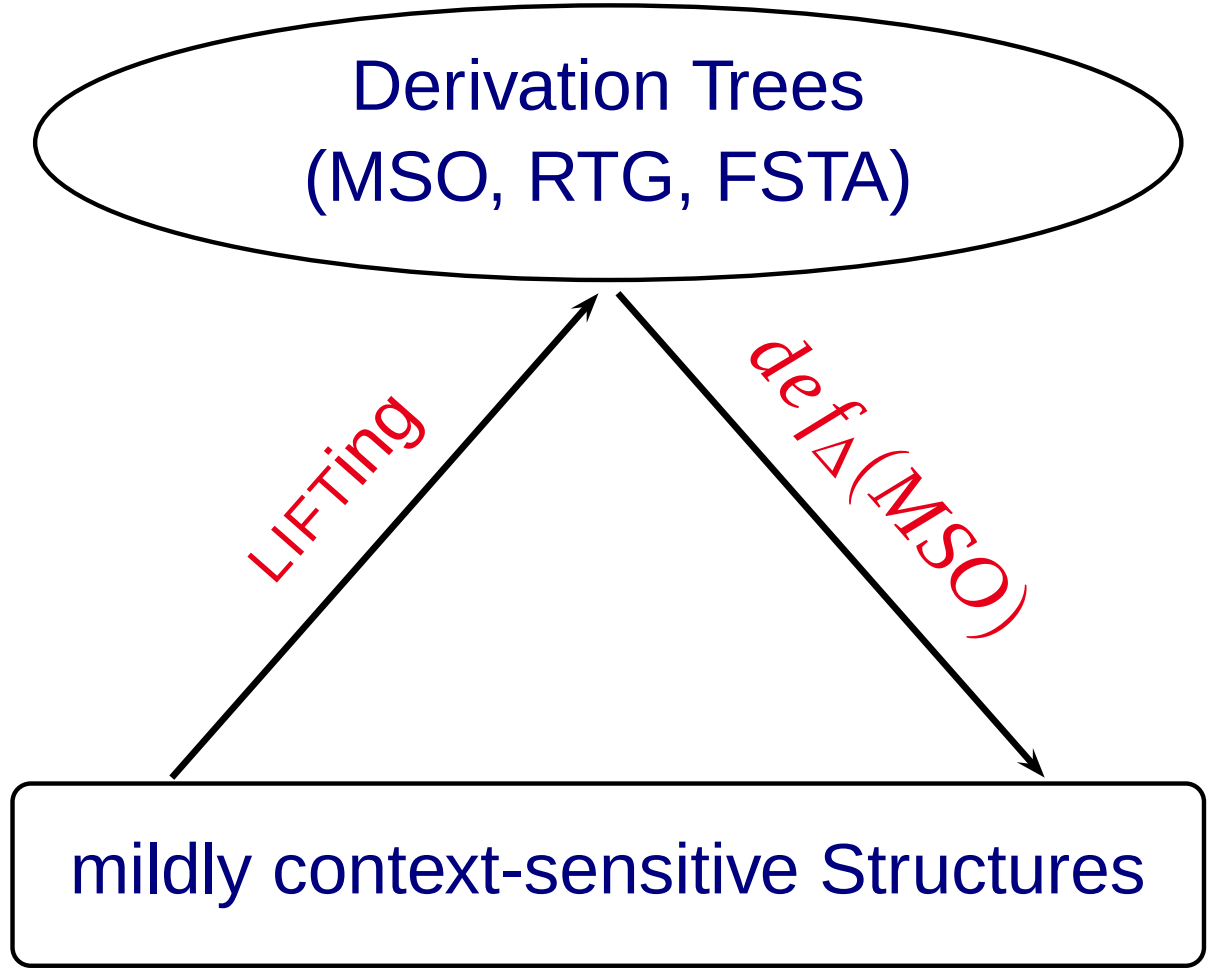
Derivation Trees
(MSO, RTG, FSTA)

mildly context-sensitive Structures

Overview Two-Steps



Overview Two-Steps



Lifting

Lifting: Intuition

- Is a simple primitive recursive function.
- Makes control structure visible.
- All function symbols become constants.
- Resulting grammar is *regular*.
- Therefore expressible by an MSO-formula, and
- there is a tree automaton for the lifted grammar.

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- Resulting grammar is *regular*.
- Therefore expressible by an MSO-formula, and
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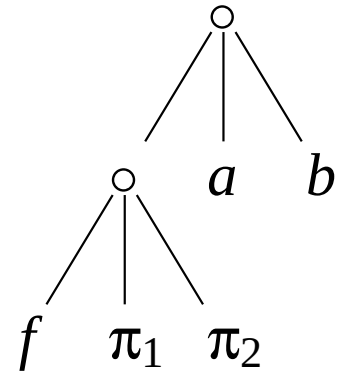
$$f(a,b) \rightsquigarrow (f \circ (\pi_1, \pi_2)) \circ (a,b) \rightsquigarrow$$



Lifting

For $k \geq 0$, $\text{LIFT}_k^{\Sigma} : T(\Sigma, \mathbf{X}_k) \rightarrow T(\Sigma^L, k)$ is defined as follows:

$$\text{LIFT}_k^{\Sigma}(x_i) = \pi_i^k$$

$$\text{LIFT}_k^{\Sigma}(f) = c_{(0,k)}(f') \text{ f"ur } f \in \Sigma_0$$

$$\text{LIFT}_k^{\Sigma}(f(t_1, \ldots, t_n)) = c_{(n,k)}(f', \text{LIFT}_k^{\Sigma}(t_1), \ldots, \text{LIFT}_k^{\Sigma}(t_n)) \text{ f"ur } f \in \Sigma_n, n \geq 1$$

The lifted Regular Tree Grammar: $\mathcal{G}'$

$$\Sigma_0^L = \{\varepsilon, a', b', c', d', S'\} \qquad \Sigma_2^L = \{\bullet'\}$$

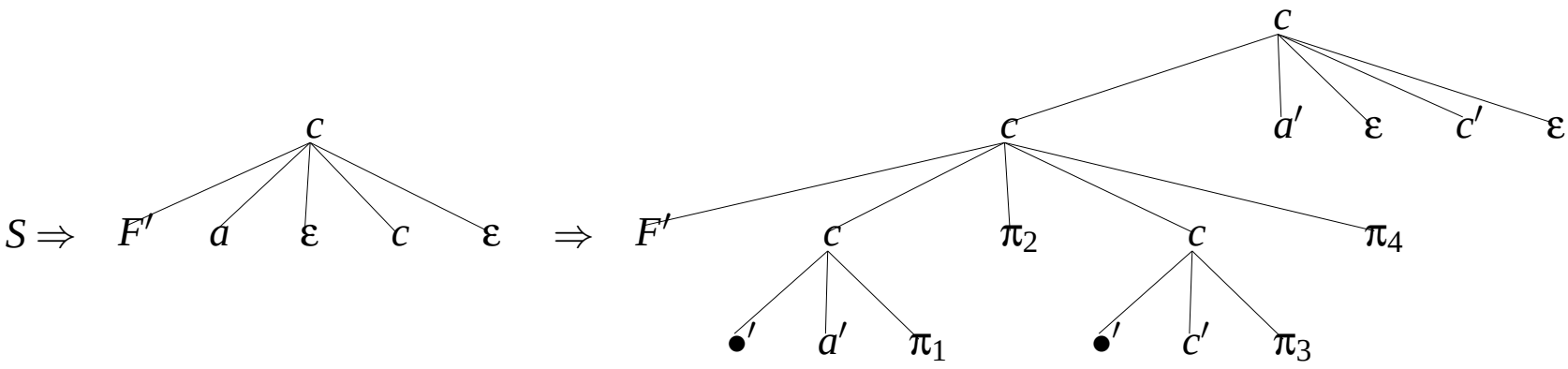
$$\Sigma_4^L = \{F'\} \cup \{\pi_1, \pi_2, \pi_3, \pi_4\} \qquad \Sigma_{n,k}^L = \{c\} \text{ for simplicity}$$

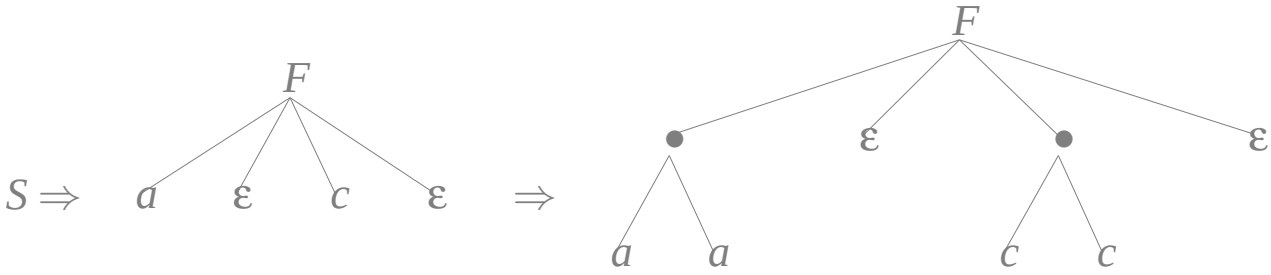
$$\mathbf{F}_0 = \{ S', F' \} \qquad S = S' \qquad \mathbf{X} = \emptyset$$

$$\mathbf{P} = \left\{ \begin{array}{lcl} S' & \longrightarrow & \varepsilon \\ S' & \longrightarrow & c(F', a', \varepsilon, c', \varepsilon) \\ S' & \longrightarrow & c(F', \varepsilon, b', \varepsilon, d') \\ F' & \longrightarrow & c(F', c(\bullet', a', \pi_1), \pi_2, c(\bullet', c', \pi_3), \pi_4) \\ F' & \longrightarrow & c(F', \pi_1, c(\bullet', b', \pi_2), \pi_3, c(\bullet', d', \pi_4)) \\ F' & \longrightarrow & c(\bullet', c(\bullet', c(\bullet', \pi_1, \pi_2), \pi_3), \pi_4) \end{array} \right\}$$

$F(x_1, x_2, x_3, x_4) \longrightarrow F(\bullet(a, x_1), x_2, \bullet(c, x_3), x_4)$

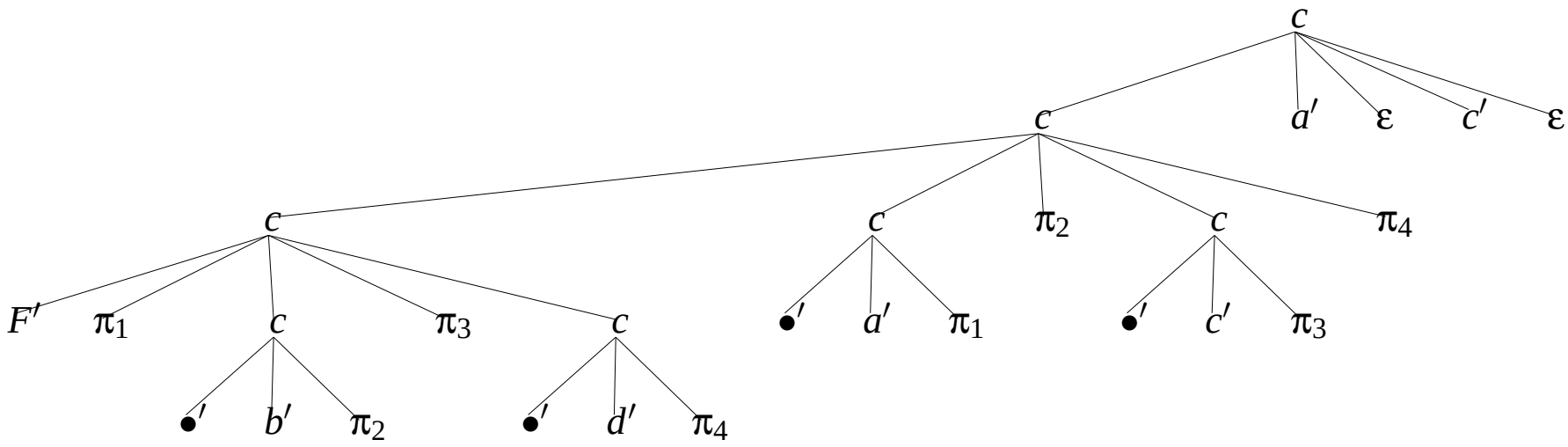
A sample derivation of $\mathcal{G}'$



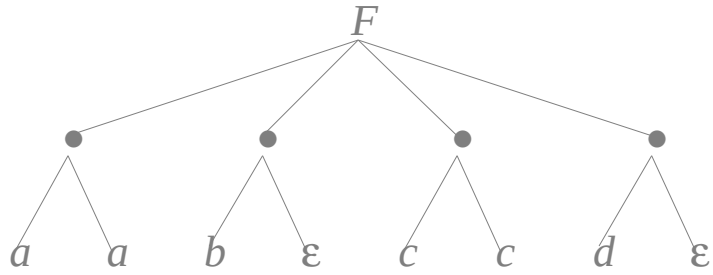


A sample derivation of $\mathcal{G}'$

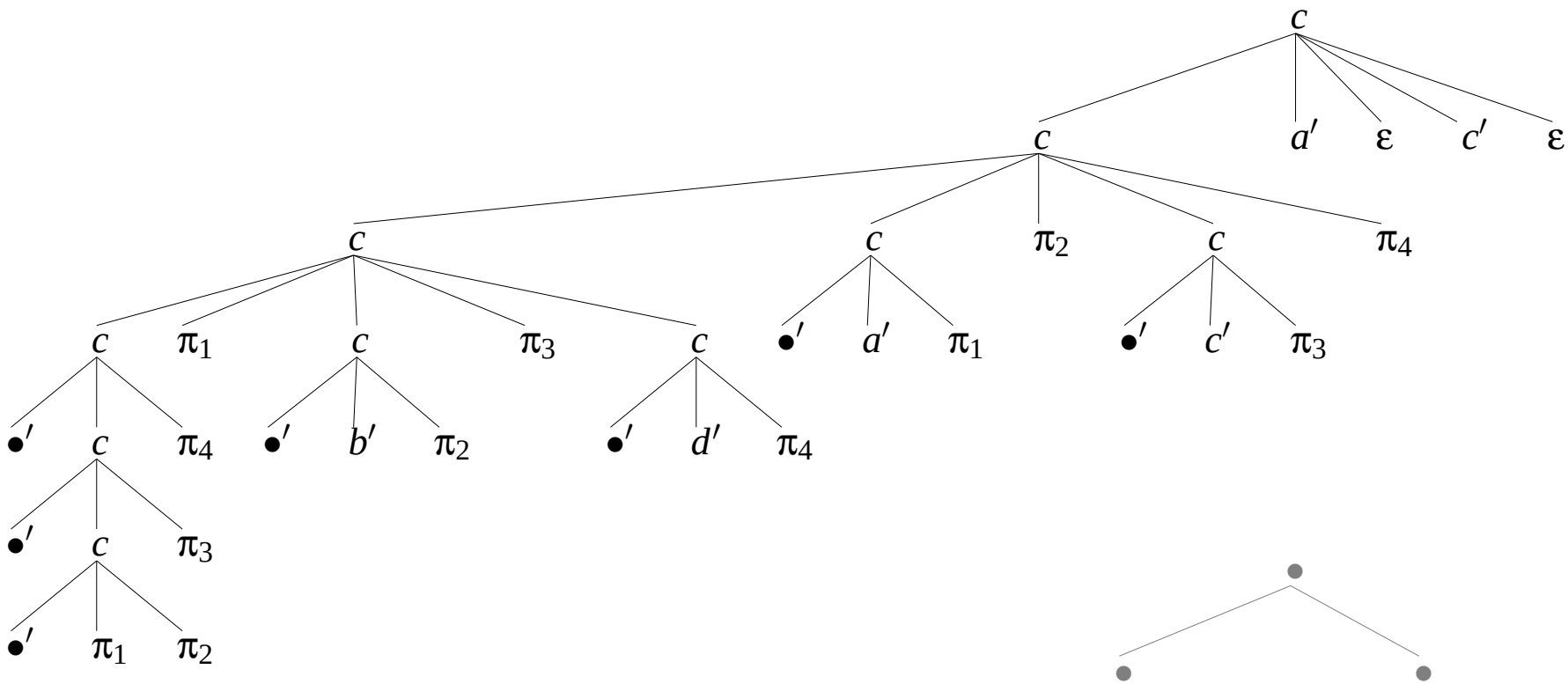
$\Rightarrow$

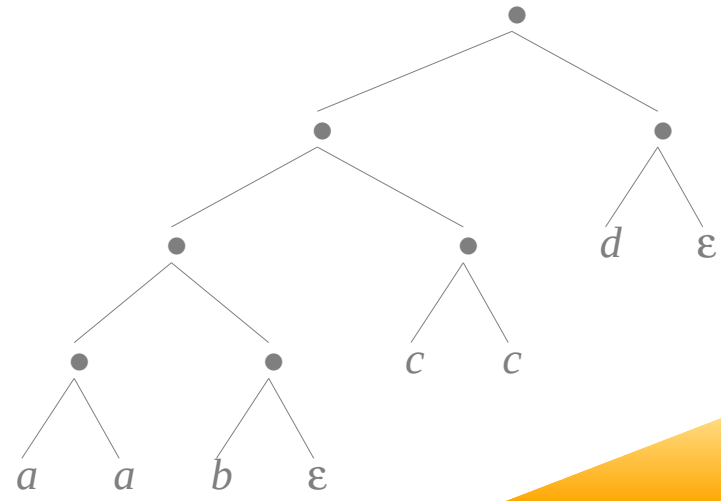


$\Rightarrow$



A sample derivation of $\mathcal{G}'$





Lifted Query

$$Q^L = \{t^L \models \varphi_r^L \wedge \psi\}$$

$\varphi_r^L \overset{def}{\Longleftrightarrow}$ "Lifted" φ_r

$\psi \overset{def}{\Longleftrightarrow}$ "Lifted" specification of cross-serial dependencies

But: the resulting trees are not in the format of the treebank and for humans/linguists unreadable.

Reconstruction

Goal of the Reconstruction

- The intended tree is contained in the lifted tree.
- Homomorphism:

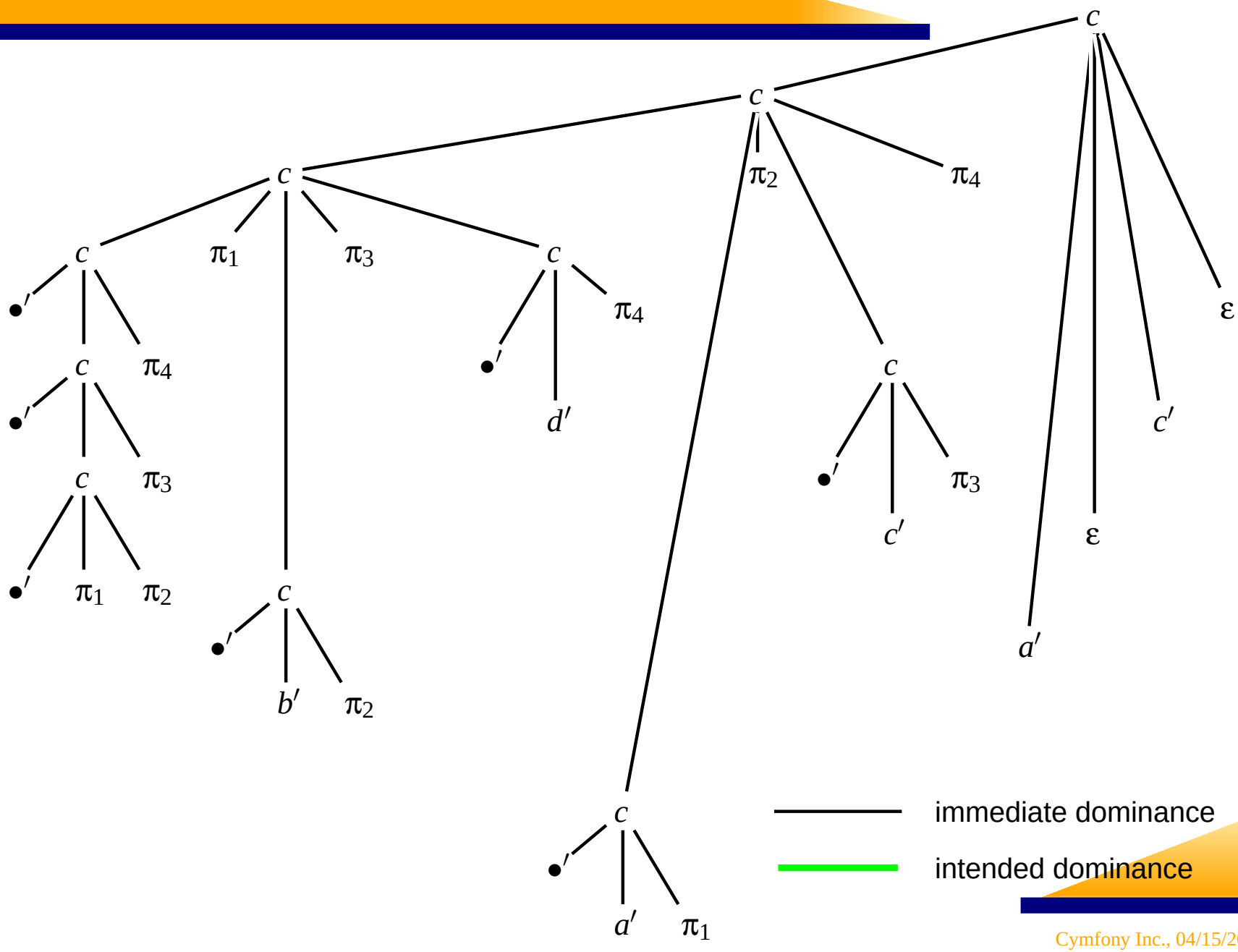
$$h(f') = f(x_1, \ldots, x_n) \text{ for } f \in \Sigma_n$$
$$h(\pi_i^n) = x_i$$
$$h(c_{(n,k)}(t, t_1, \ldots, t_n)) = h(t)[h(t_1), \ldots, h(t_n)]$$

- Definition of a set of intended relations:

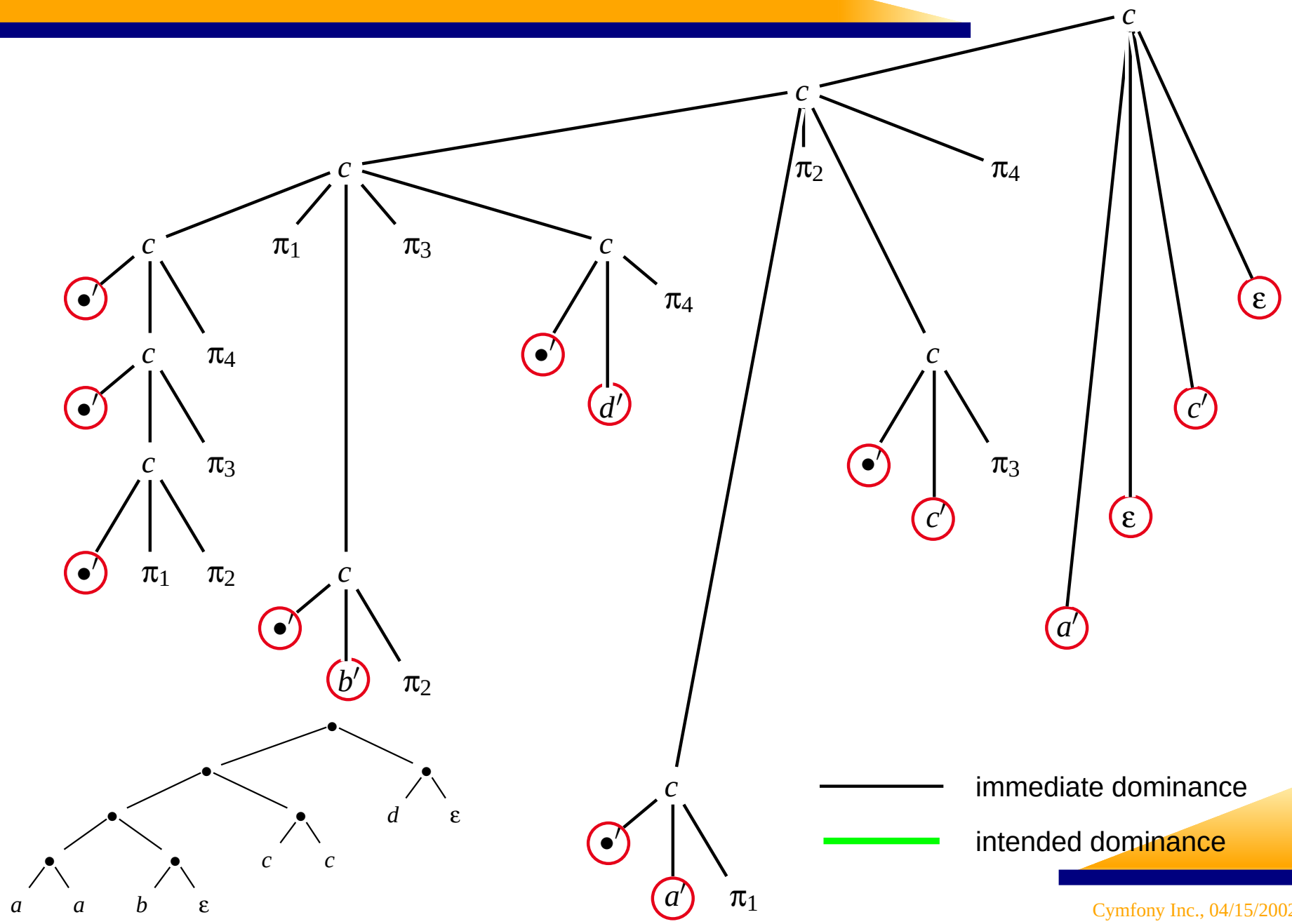
$$R^I = \{\lhd^\star \text{ (dominance)}, \lhd, \lhd^+, \lhd \text{ (precedence)}, \ldots\}$$

➪ MSO-Transduction or Macro-Tree Transducer

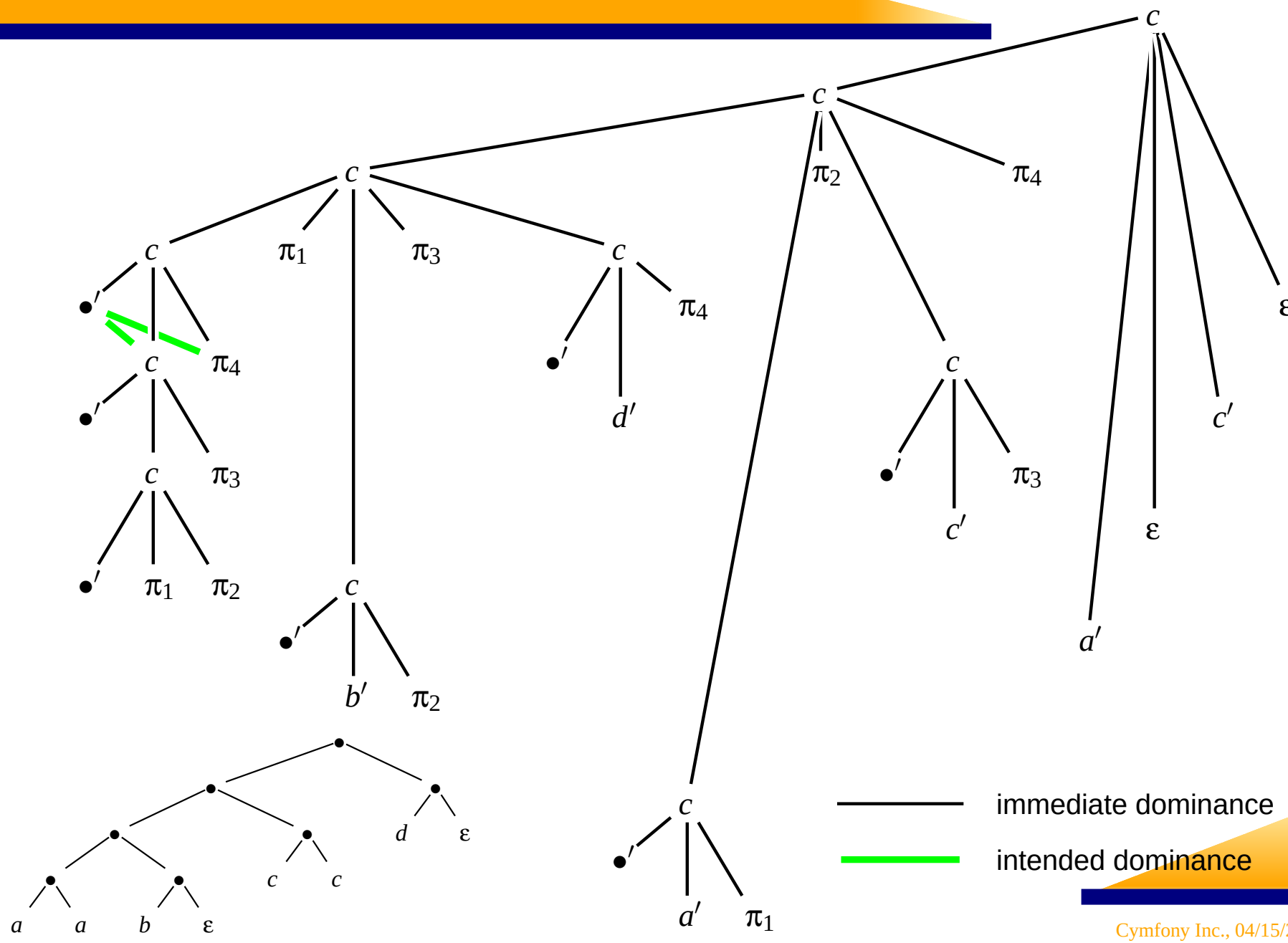
Reconstruction of the Intended Structures



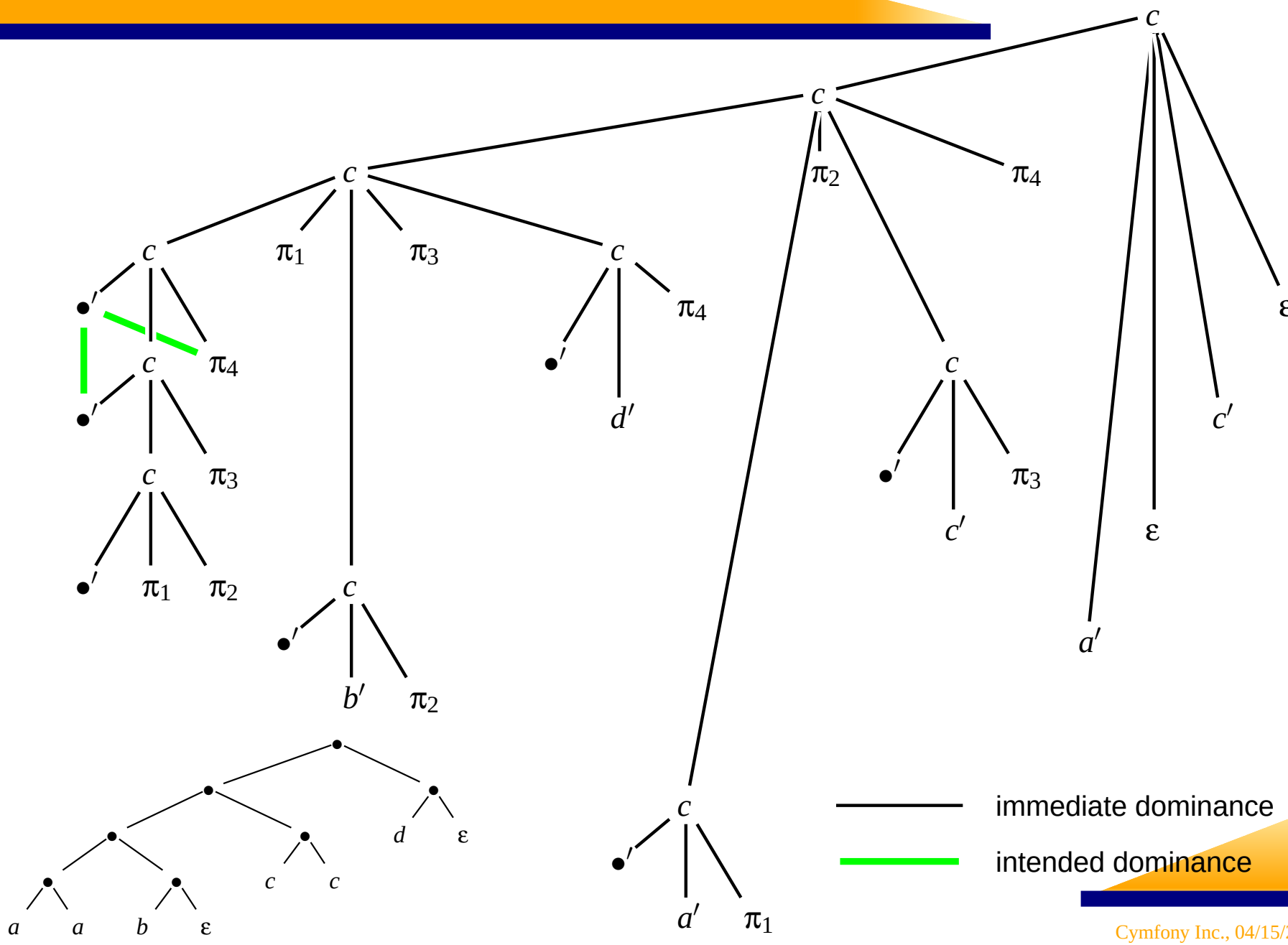
Reconstruction of the Intended Structures



Reconstruction of the Intended Structures



Reconstruction of the Intended Structures



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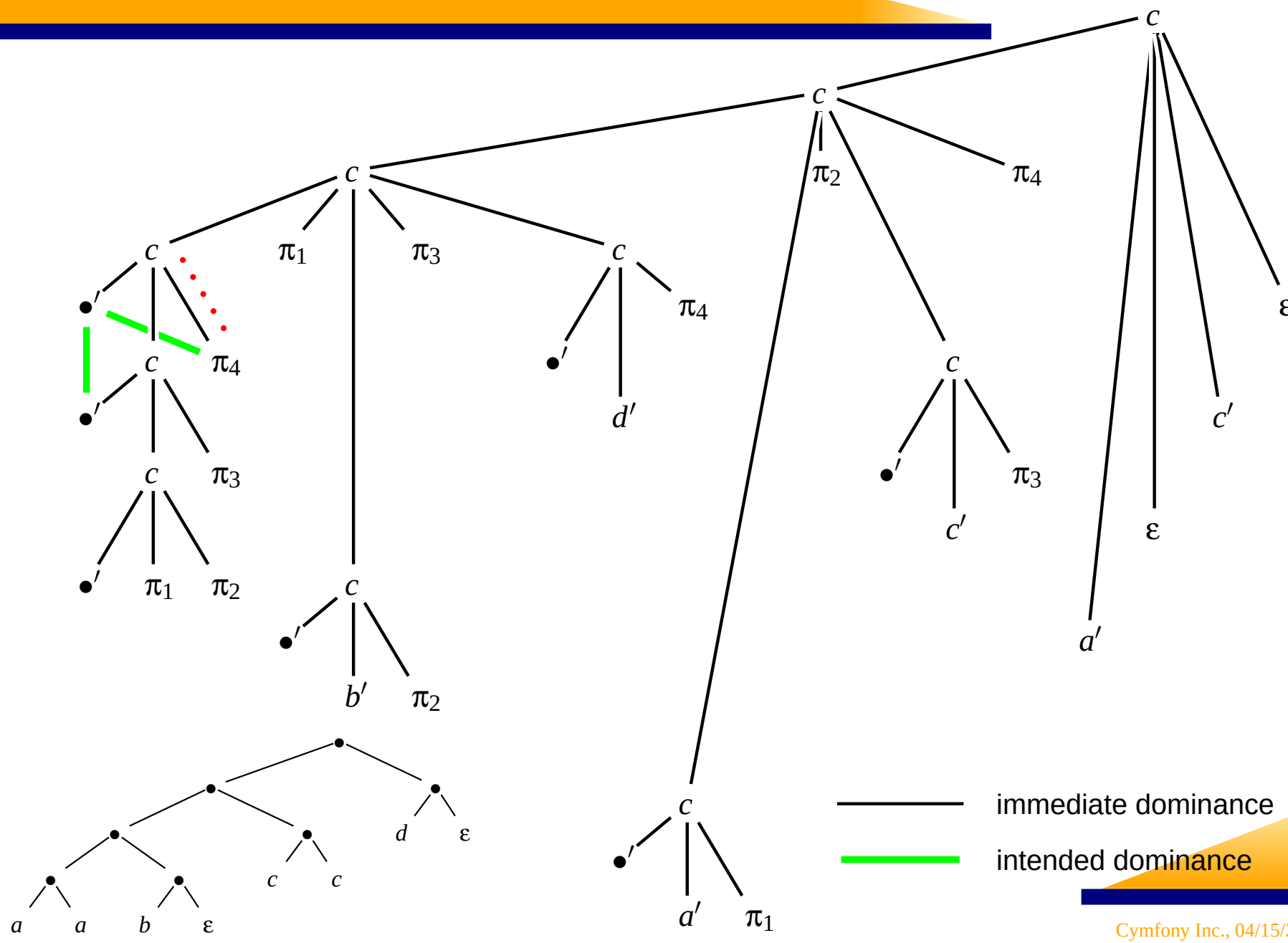

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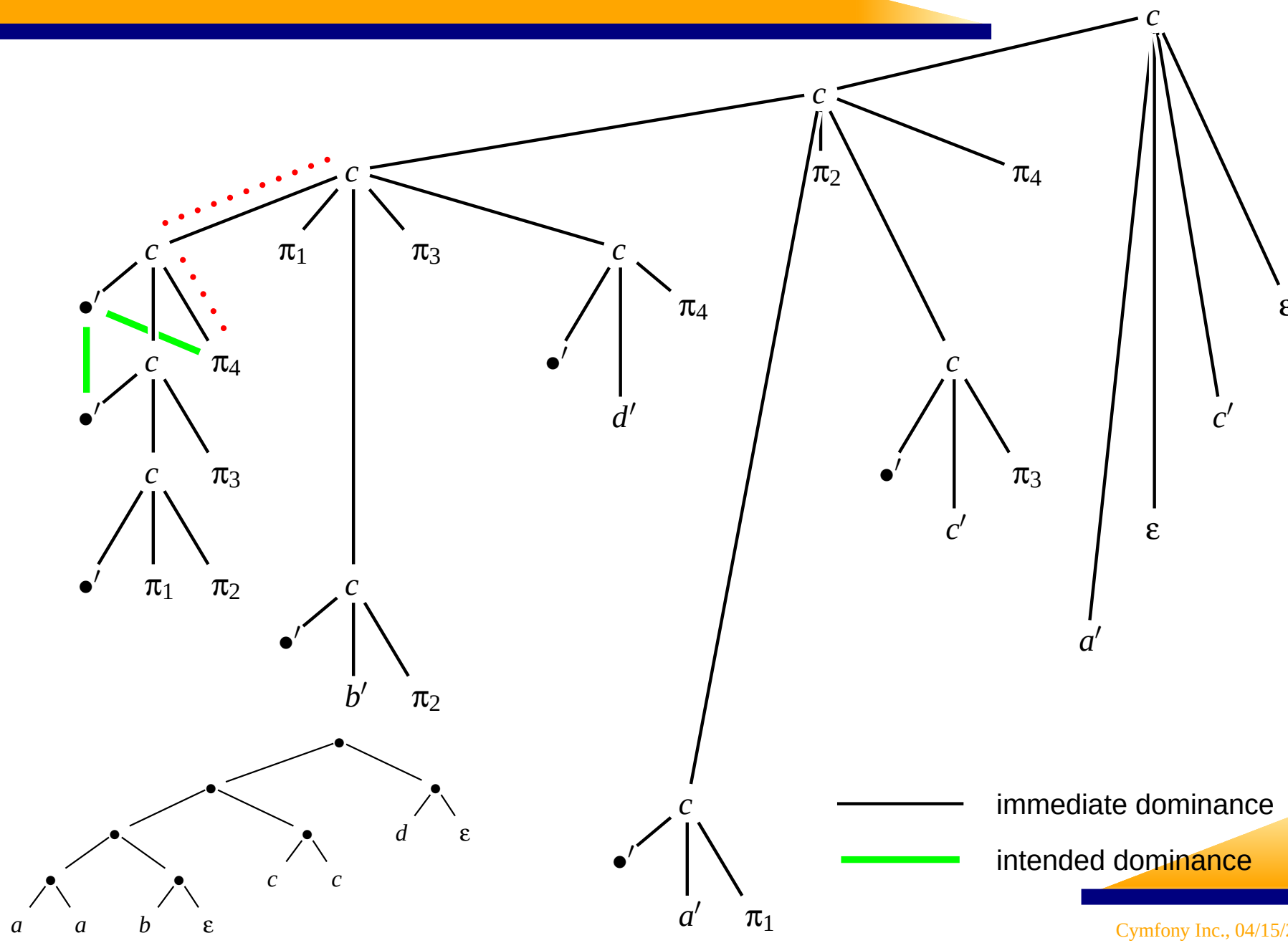

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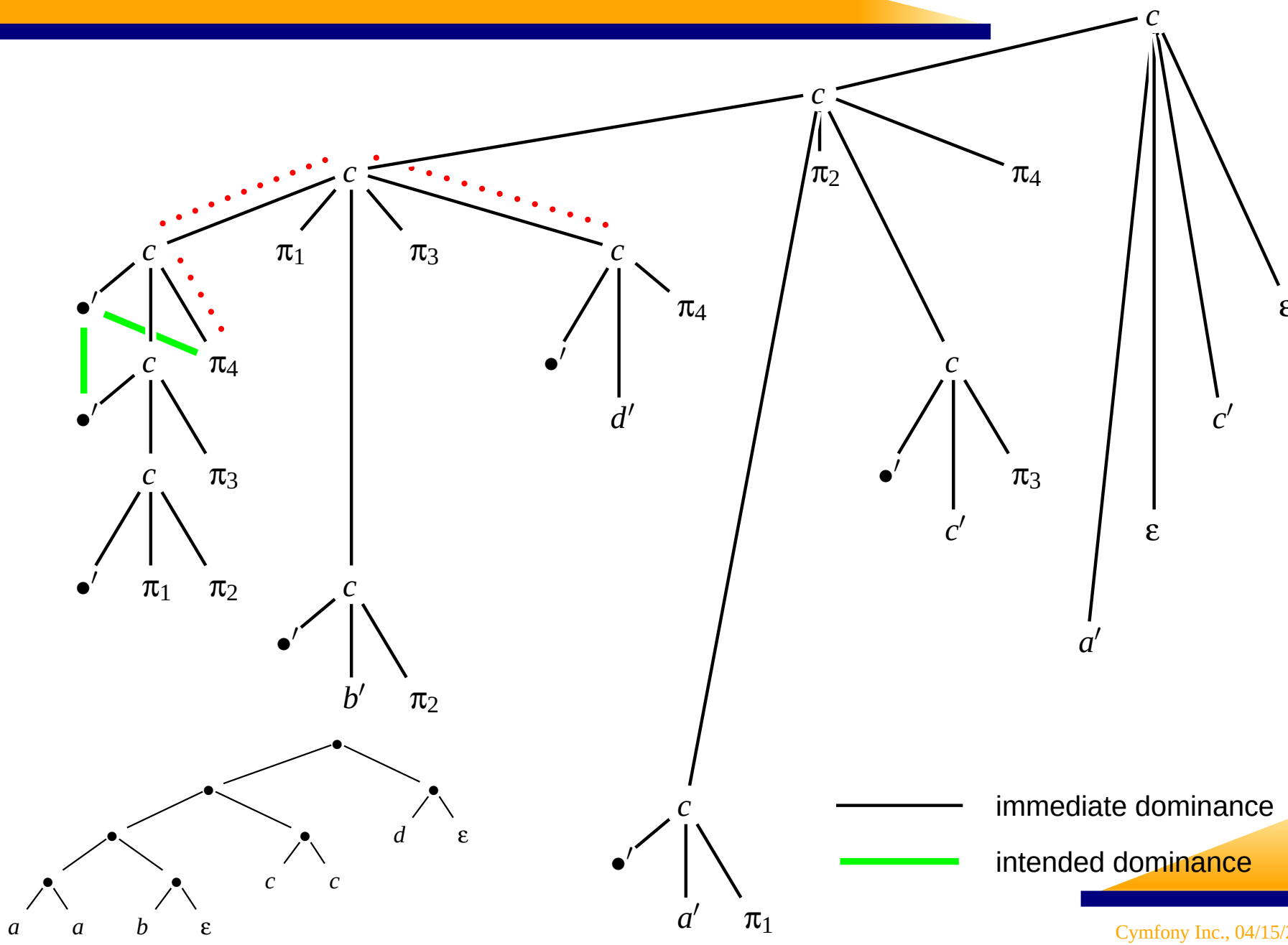

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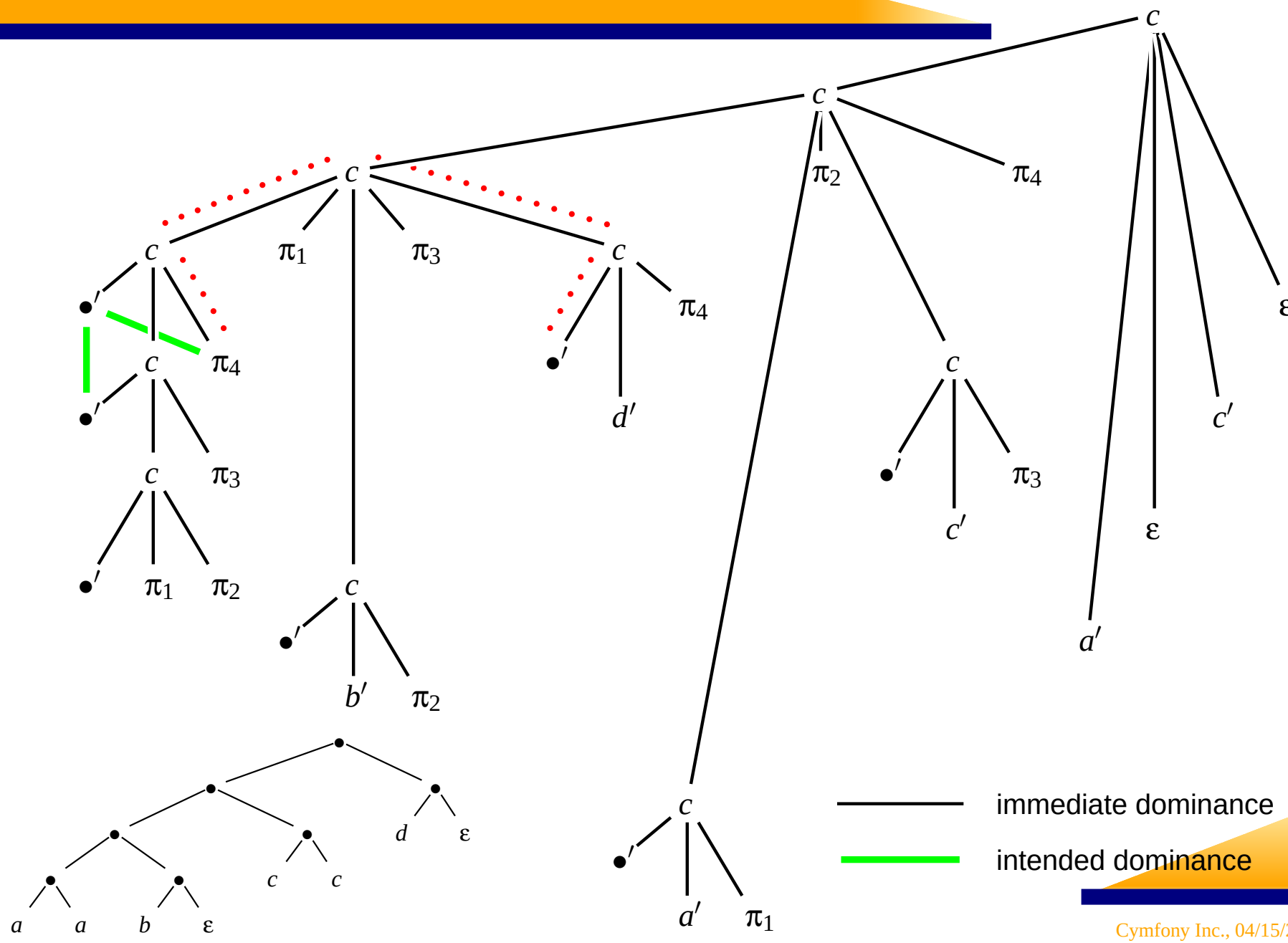

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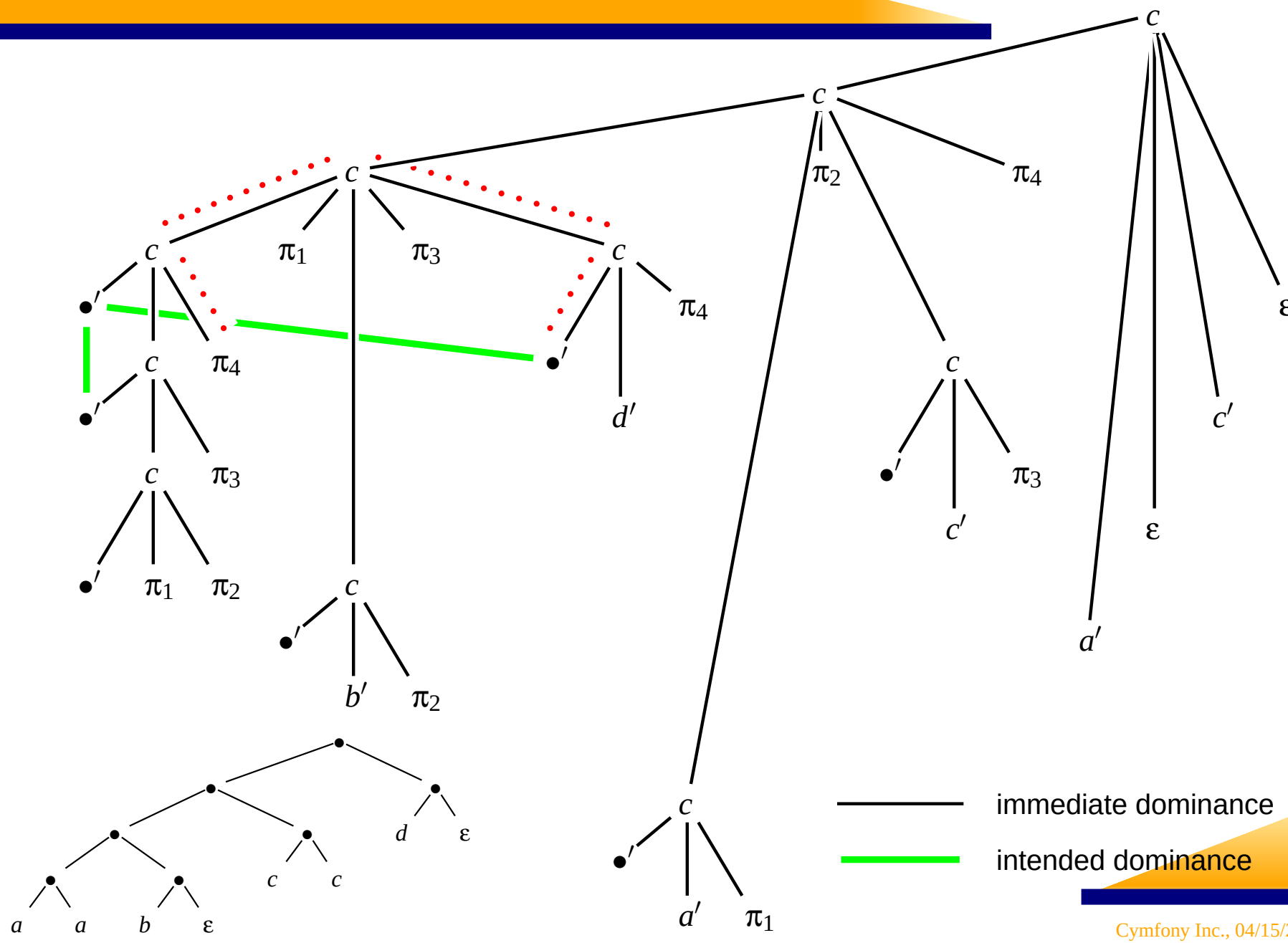

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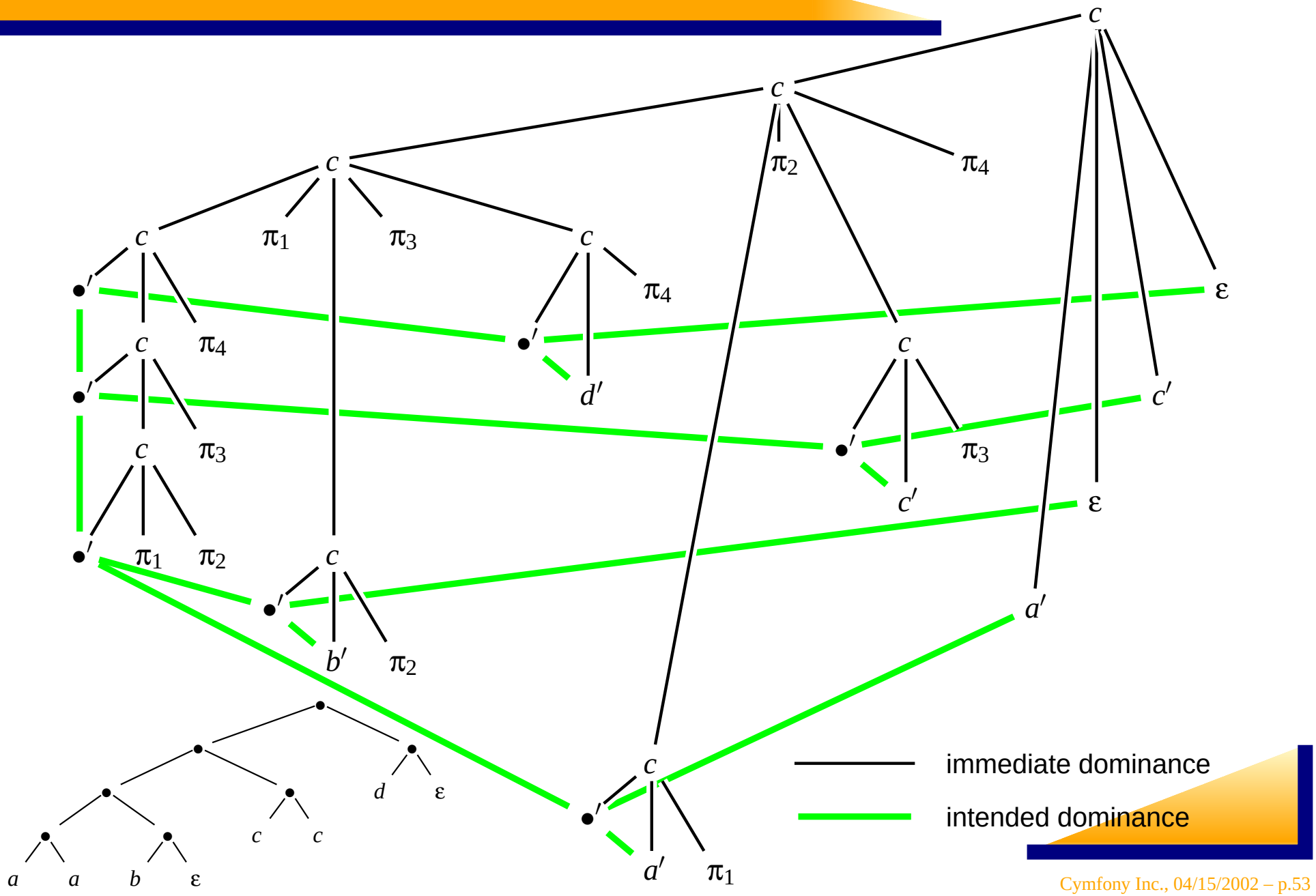
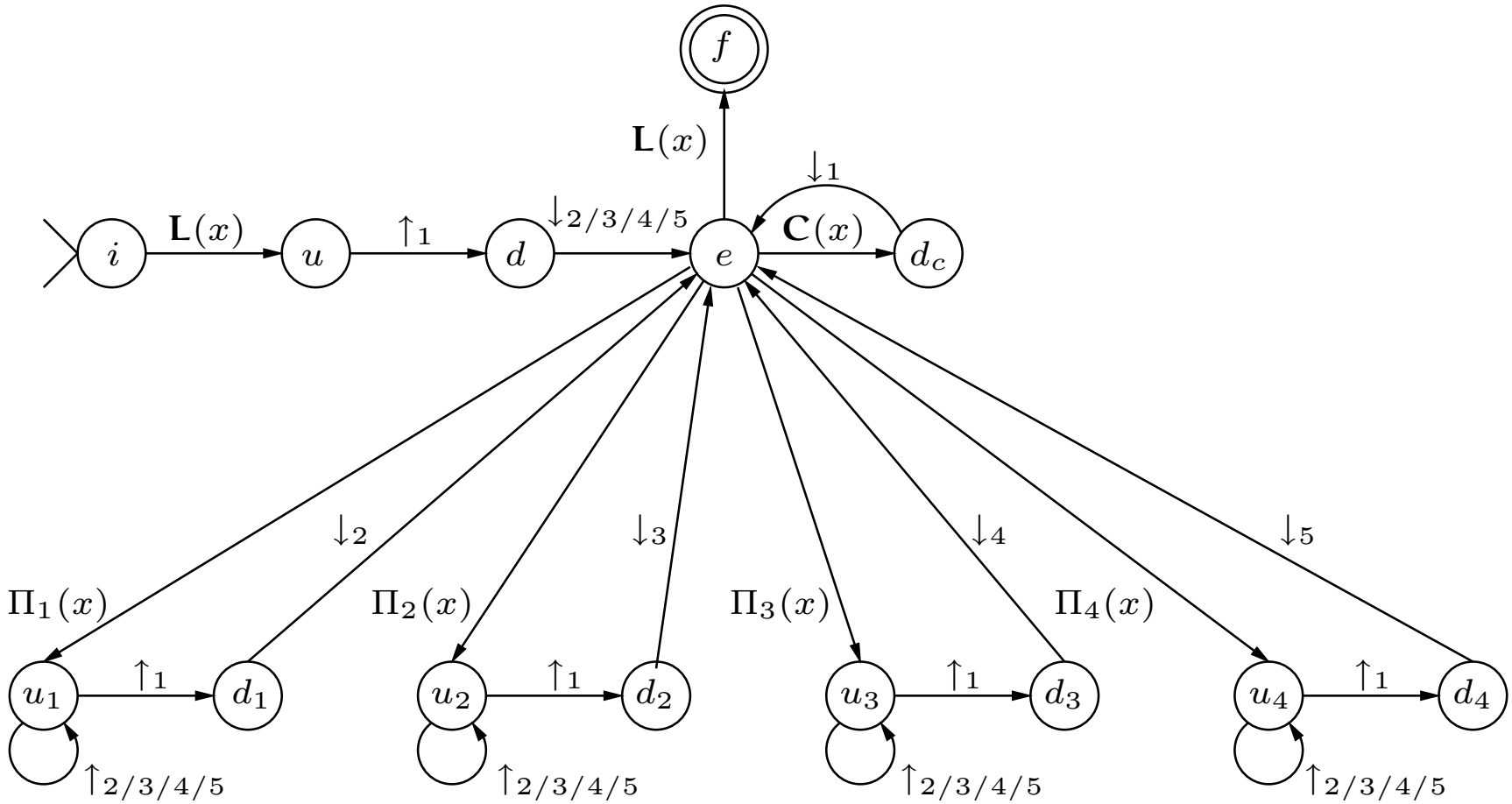


The tree-walking automaton $\mathfrak{A}$



Regular Tree-Node Relations

For any tree t, such a tree-walking automaton $\mathfrak{A}$ computes a node relation

$$R_t(\mathfrak{A}) = \{(x,y) \mid (x,q_i) \stackrel{*}{\Longrightarrow} (y,q_f) \text{ for some } q_i \in I \text{ and some } q_f \in F\}$$

where for all states $q_i, q_j \in Q$ and nodes x, y in t

$$(x,q_i) \Rightarrow (y,q_j) \text{ iff } \exists d \in \Delta : (q_i,d,q_j) \in \delta$$

and y is reachable from x in t via d.

If all the tests $\phi(x)$ of $\mathfrak{A}$ are MSO definable, $\mathfrak{A}$ specifies a *regular tree-node relation*, which is itself MSO definable.

MSO definable Transduction

$$\mathcal{R} \rightsquigarrow \mathcal{S}$$

$$\Delta = (\chi, \xi, (\theta_s)_{s \in \mathcal{S}})$$

χ	the domain of the transduction
ξ	the resulting domain of $\mathcal{S}$
θ_s	the new relations

Definition by Courcelle 1997

Basic Idea Rabin 1965

Intended Structures via MSO transduction

$$(\varphi, \psi, (\theta_s)_{s \in S})$$

$$S = \{\blacktriangleleft, \blacktriangleleft^\star, \blacktriangleleft^+, \triangleleft, \dots\}$$

$$\chi = \varphi_r^L \wedge \psi$$

$$\xi = \text{nodes with a “linguistic” label}$$

$$\theta_{\blacktriangleleft} = \text{trans}_{W_{\blacktriangleleft}}(x,y)$$

$$\theta_{\blacktriangleleft^\star} = (\forall X)[\blacktriangleleft\text{-}\mathit{closed}(X) \wedge x \in X \rightarrow y \in X]$$

$$\theta_{\blacktriangleleft^+} = x \blacktriangleleft^\star y \vee x \not\approx y$$

$$\theta_{\triangleleft} = \text{trans}_{W_{\triangleleft}}(x,y)$$

$$\theta_{\text{label}} = \text{ taken from } R$$

Top-Down Tree Transducer

A top-down tree transducer (TDTT) is a a tuple $T = \langle Q, \Sigma, \Omega, q_0, P \rangle$ with states Q, ranked alphabets Σ and Ω (input and output), initial state q_0 and a finite set of productions P of the form

$$q(\sigma(x_1, \ldots, x_n)) \longrightarrow t$$

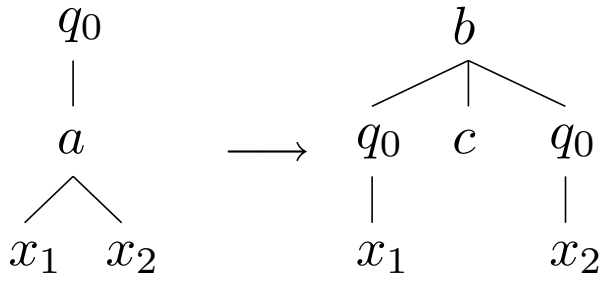
where $n \geq 0$, $\sigma \in \Sigma_n$ and $t \in T(\Omega \cup \Sigma(X) \cup Q)$.

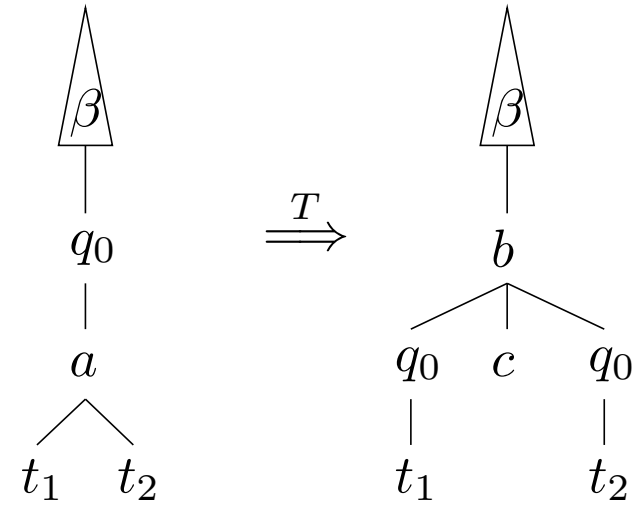
The transition relation ($\overset{T}{\Longrightarrow}$) is defined as usual. The transduction realized by a top-down tree transducer T is then defined to be $\{(t_1, t_2) \in T(\Sigma) \times T(\Omega) \mid q_0(t_1) \overset{T}{\Longrightarrow}^* t_2\}$.

Top-Down Tree Transducer

$$q_0(a(x_1, x_2)) \longrightarrow b(q_0(x_1), c, q_0(x_2))$$

$$q_0(p) \longrightarrow q$$





Macro Tree Transducer

A macro tree transducer (MTT) is a five-tuple $M = \langle Q, \Sigma, \Omega, q_0, P \rangle$ with Q a ranked alphabet of states, ranked alphabets Σ and Ω (input and output), initial state q_0 of rank 1, and a finite set of productions P of the form

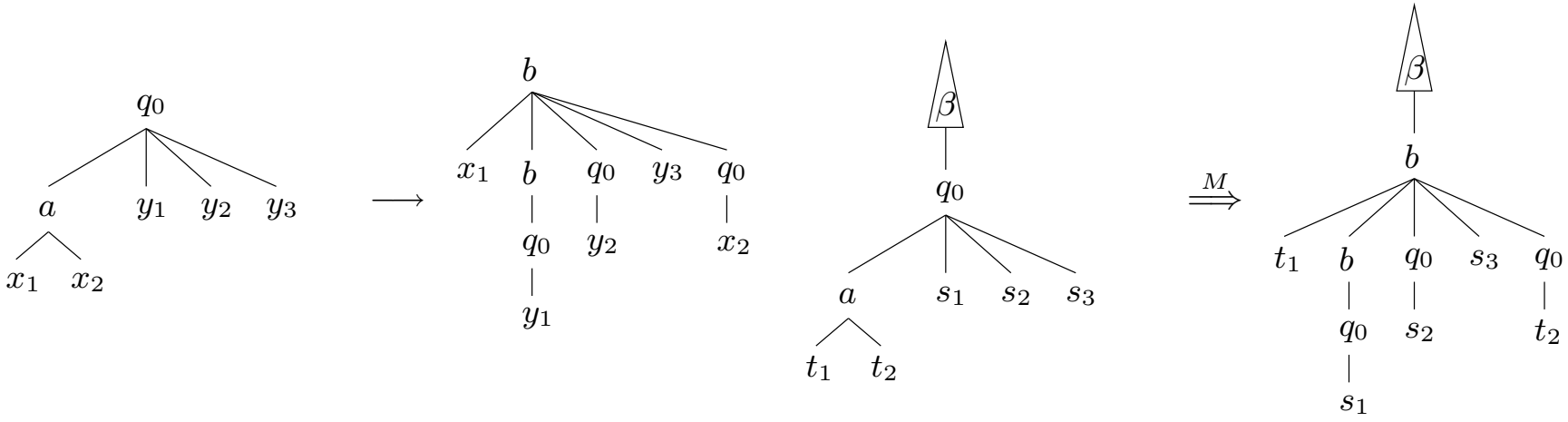
$$q(\sigma(x_1, \ldots, x_n), y_1, \ldots, y_m) \longrightarrow t$$

where $n, m \geq 0$, $q \in Q_{m+1}$, $\sigma \in \Sigma_n$ and $t \in RHS(\Sigma, \Omega, n, m)$.

The productions $p \in P$ of M are used as term rewriting rules in the usual way. The transition relation of M is denoted by $\overset{M}{\Longrightarrow}$. The transduction realized by M is the function $\{(t_1, t_2) \in T(\Sigma) \times T(\Omega) \mid (q_0, t_1) \overset{M}{\Longrightarrow}^* t_2\}$.

Macro Tree Transducer

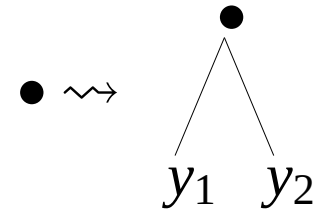
$$q_0(a(x_1,x_2),y_1,y_2,y_3) \longrightarrow b(x_1,b(q_0(y_1)),q_0(y_2),y_3,q_0(x_2))$$

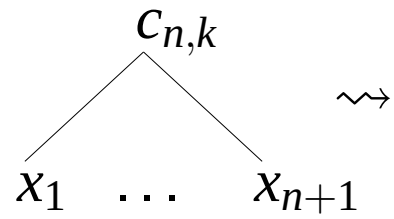


Constructing an MTT: Intuition

$$\sigma \rightsquigarrow \sigma$$

$$\pi_i \rightsquigarrow y_i$$

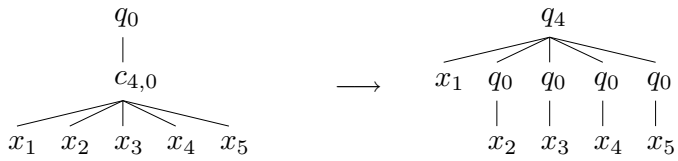


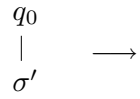


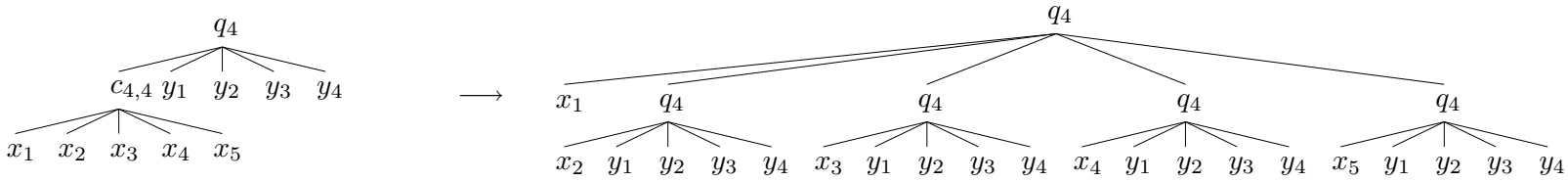
$$q_k(\underbrace{c_{n,k}(x_1,\ldots,x_{n+1}),y_1,\ldots,y_k}_{k+1}) \longrightarrow$$

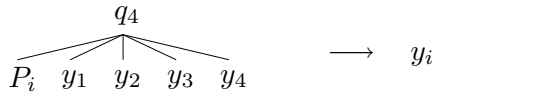
$$q_n(\underbrace{x_1,q_k(x_2,y_1,\ldots,y_k),\ldots,q_k(x_{n+1},y_1,\ldots,y_k)}_{n+1})$$

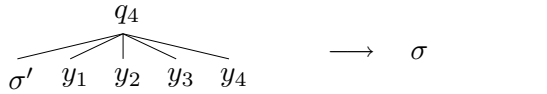
The Example MTT for $\mathcal{G}'$

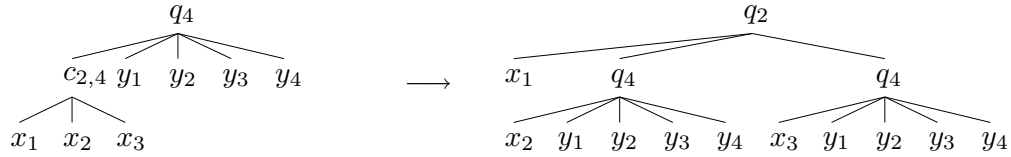












Intended Query-Results

$$Q' = \{def_\Delta(t^L) \mid t^L \models \varphi_r^L \wedge \psi\}$$

$def_\Delta \overset{def}{\Longleftrightarrow}$ transduction from the domain of the lifted regular trees into original tree data base

$\Delta \overset{def}{\Longleftrightarrow}$ MSO definable transduction

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Summary and Outlook

Summary

- Querying context-sensitive relations with regular means.
- three “kinds” of trees
  - trees as elements of a free algebra
  - trees as elements of a free clone (Lawvere-Algebra)
  - trees as relational structures
- transduction is effectively realized via tree-walking automata or macro tree transducer

Outlook

- unranked trees
- implementation
- efficiency
  - MSO decidable on trees
  - MSO undecidable on graphs
  - MSO + one binary relation undecidable on trees
  - MSO → tree automata hyper-exponential
  - “Guarded” MSO → tree automata exponential
  - Lifting and transduction require linear time
  - Lifting of the candidate trees?

Overview

- Introduction and Motivation
- Semistructured Data
- Query Languages
- Intermediate Conclusion
- Secondary Edges
- Regular Queries of Context-Sensitive Relations
- MSO Queries
- Lifting
- Reconstruction
- Conclusion and Outlook

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Appendix
Some Details and Definitions

MSO Logic on Trees (Rogers, 1994)

- K a set of individual constants
- P a set of predicate constants
- immediate dominance: $\lhd$
- reflexive dominance: $\lhd^*$
- proper dominance: $\lhd^+$
- proper precedence: $\prec$
- a set of *monadic* second-order variables ($X, Y, \ldots$);
- a set of individual variables ($x, y, \ldots$);
- boolean connectives: $\neg$, $\wedge$, $\ldots$;
- Quantifiers over individuals and sets: $\exists$, $\forall$;
- equality and membership: $\approx$, $\in$.

Example formulas

$$X \subseteq Y \stackrel{def}{\Longleftrightarrow} (\forall x)[x \in X \Rightarrow x \in Y]$$

$$\text{Sing}(X) \stackrel{def}{\Longleftrightarrow}$$

% All subsets of X are equal to X or empty:

$$(\forall Y)[Y \subseteq X \Rightarrow (X \subseteq Y \vee (\forall Z)[Y \subseteq Z])] \wedge$$

% X is not the empty set:

$$(\exists Y)[X \not\subseteq Y]$$

Example formulas

AC-Com$(x,y) \stackrel{def}{\Longleftrightarrow}$

% x c-commands y:

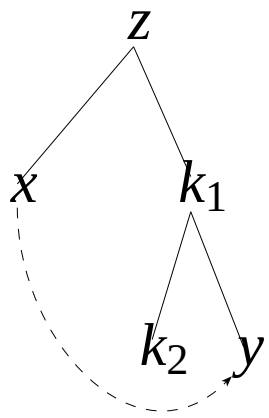
$(\forall z)[z \lhd^{+} x \Rightarrow z \lhd^{+} y] \wedge \neg(x \lhd^{*} y) \wedge$

% y does not c-command x:

$\neg((\forall z)[z \lhd^{+} y \Rightarrow z \lhd^{+} x] \wedge \neg(y \lhd^{*} x)) \wedge$

% x precedes y:

$x \prec y$



Example formulas

$$\mathrm{LC}(x,y) \stackrel{def}{\Longleftrightarrow}$$

$$\mathrm{AC\text{-}Com}(x,y) \wedge$$

% There is no k_1 with property P:

$$(\neg\exists k_1)[k_1 \in P \wedge$$

% such that it intervenes between x and y:

$$(\exists z)[z \lhd x \wedge z \lhd^{+} k_1 \wedge k_1 \lhd^{+} y]]$$

Properties of MSO Logic on Trees

- Advantages:
  - Concise Representation in a Powerful and Flexible Logic
  - Descriptive Complexity Results
  - Operational and Denotational Semantics
  - Decidable
  - Compilation: Formulas to Tree Automata
- Shortcomings
  - Non-Elementary Complexity of the Decidability Algorithm:

$$|A| = 2^{2^{\cdot^{\cdot^{\cdot^{2^n}}}}} \Big\} m$$

  - Descriptive Complexity: Only Context-Free String Languages

“Linguistic” Definitions

- A constraint is **satisfiable** iff it results in a non-empty automaton.
- A constraint is **valid** iff it results in the trivial non-empty automaton.
- A (maximally) binary branching **tree** is a subset T of the domain of MSO Logic on Trees. A **labeled tree** is a tuple $\langle T, F_1, \ldots, F_k \rangle$.
- A **grammar** in this setting is a definition of a $k+1$-ary relation in MSO Logic on Trees designating all and only the well-formed labeled trees.
- The **recognition problem** is the determinization of the emptiness of the conjunction of the grammar formula/automaton with the input formula/automaton.
- The **parsing problem** is the conjunction of the grammar formula/automaton with the input formula/automaton.

Coding RTGs in MSO Logic (Thomas 1997)

- Define a tree automaton $\mathfrak{A}_{\mathcal{G}'}$ from the RTG $\mathcal{G}'$
- Code its behaviour in $\varphi_{\mathfrak{A}_{\mathcal{G}'}}$

$$(\exists X_0, \ldots, X_m)[\bigwedge_{i \neq j} (\neg\exists y)[y \in X_i \wedge y \in X_j] \wedge$$

$$(\forall x)[(\neg\exists y)[x \lhd y] \rightarrow x \in X_0] \wedge \qquad \text{\% Initial State}$$

$$\bigwedge_{1 \leq n \leq m} (\forall x_1, \ldots, x_n, y)[\bigvee_{\substack{(i_1,\ldots,i_n,\sigma,j) \in \alpha \\ 1 \leq k \leq n}} x_k \in X_{i_k} \wedge y \lhd x_k \wedge y \in X_j \wedge y \in P_\sigma]$$

$$\bigvee_{i \in F} (\exists x \forall y)[x \lhd^* y \wedge x \in X_i] \qquad \text{\% Root}$$

Lifting

Suppose that Σ is a ranked alphabet. The *derived* $\mathbb{N}$-sorted alphabet Σ^L is defined as follows:

For each $n \geq 0$,

$$\Sigma'_n = \{ f' \mid f \in \Sigma_n \}$$

is a new set of symbols of sort n;

back to Lifting

Lifting

Suppose that Σ is a ranked alphabet. The *derived* $\mathbb{N}$-sorted alphabet Σ^L is defined as follows:

for each $n \geq 1$ and each $i, 1 \leq i \leq n$,

$$\pi_i^n$$

is a new symbol, the ith projection symbol of sort n;

back to Lifting

Lifting

Suppose that Σ is a ranked alphabet. The *derived* $\mathbb{N}$-sorted alphabet Σ^L is defined as follows:

for each $n \geq 0, k \geq 0$ the new symbol

$$c_{(n,k)}$$

is the (n,k)th composition symbol.

back to Lifting

Lifting

Suppose that Σ is a ranked alphabet. The *derived* $\mathbb{N}$-sorted alphabet Σ^L is defined as follows:

$$\Sigma^L_0 = \Sigma'_0$$
$$\Sigma^L_n = \Sigma'_n \cup \{\pi^n_i | 1 \leq i \leq n\} \text{ for } n \geq 1$$
$$\Sigma^L_{n,k} = \{c_{(n,k)}\} \text{ for } n, k \geq 0$$
$$\Sigma^L_i = \emptyset \text{ otherwise}$$

back to Lifting

Model-Theoretic Interpretation

Basic Idea (Rabin 1965)

Obtaining a structure $\mathbb{B} = \langle B, Q \rangle$ from a structure $\mathbb{A} = \langle A, \mathcal{R} \rangle$ where $\mathcal{R}$ and Q are families of relation symbols.

back to MSO transduction

Tree-Walking Automaton

A *tree-walking automaton (with tests)* (Bloem and Engelfriet 1997) over some ranked alphabet Σ is a finite automaton $\mathfrak{A} = (Q, \Delta, \delta, I, F)$ with states Q, directives Δ, transitions $\delta : Q \times \Delta Q$ and the initial and final states $I \subseteq Q$ and $F \subseteq Q$ which traverses a tree using three kinds of directives:

- $\uparrow_i$ "move up to the mother of the current node (if it has one and it is its i-th daughter)",
- $\downarrow_i$ "move to the i-th daughter of the current node (if it exists)",
- $\phi(x)$ "verify that ϕ holds at the current node".

back to the dominance automaton

Reflexive Closure in *MSO*

$$\textit{R-closed}(X) \stackrel{def}{\Longleftrightarrow} (\forall x,y)[x \in X \wedge R(x,y) \rightarrow y \in X]$$

$$R^*(x,y) \stackrel{def}{\Longleftrightarrow} (\forall X)[\textit{R-closed}(X) \wedge x \in X \rightarrow y \in X]$$

back to the MSO transduction

Walking language $\Rightarrow$ *MSO*-formula

$$\begin{aligned}
trans_{\emptyset}(x,y) &= \bot \\
trans_{\downarrow_i}(x,y) &= edge_i(x,y) \\
trans_{\uparrow_i}(x,y) &= edge_i(y,x) \\
trans_{\phi(x)}(x,y) &= \phi(x) \wedge x = y \\
trans_{W_1 \cup W_2}(x,y) &= trans_{W_1}(x,y) \vee trans_{W_2}(x,y) \\
trans_{W_1 \cdot W_2}(x,y) &= \exists z \left(trans_{W_1}(x,z) \wedge trans_{W_2}(z,y) \right) \\
trans_{W^*}(x,y) &= trans^*_W(x,y) \\
trans^*_W(x,y) &= \forall X \left(\forall v,w \left(v \in X \wedge trans_W(v,w) \rightarrow w \in X \right) \right. \\
&\qquad \left. \wedge\, x \in X \rightarrow y \in X \right)
\end{aligned}$$

back to the MSO transduction

Complex Right-Hand-Sides

Let Σ and Ω be ranked alphabets and $n, m \geq 0$. The set of right hand sides $RHS(\Sigma, \Omega, n, m)$ over Σ and Ω with n variables and m parameters is the smallest set $rhs \subseteq T(\Sigma \cup \Omega, X_n \cup Y_m)$ such that

1. $Y_m \subseteq rhs$
2. For $\omega \in \Omega_k$ with $k \geq 0$ and $\varphi_1, \ldots, \varphi_k \in rhs$, $\omega(\varphi_1, \ldots, \varphi_k) \in rhs$
3. For $q \in Q_{k+1}$ with $k \geq 0$, $x_i \in X_n$ and $\varphi_1, \ldots, \varphi_k \in rhs$, $q(x_i, \varphi_1, \ldots, \varphi_k) \in rhs$

back to MTTs